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1. a) Convert $\frac{5\pi}{36}$ into degrees. ***25°***

b) Convert 75° into radians. $\boldsymbol{\frac{5\pi}{12}}$ ***rad***

2. A central angle measuring 54° sub-tends an arc on the circle.
If the circle has a radius of 12 cm, calculate the length of the sub-tended arc (round to the nearest tenth). ***11.3 cm***

3. What are the coordinates of the trigonometric point P(t) located in the 3rd quadrant if $\cos t = -\frac{15}{17}$? $P(t) = \left(\frac{-15}{17}, \frac{-8}{17}\right)$

4. Evaluate $\sin\left(t - \frac{\pi}{4}\right)$ if $\frac{\pi}{2} < t < \pi$ and $\sin t = \frac{\sqrt{3}}{2}$.

$\cos t = -\frac{1}{2}$

$\sin\left(t - \frac{\pi}{4}\right) = \sin t \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos t = \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

6. Find the coordinates of the point P(t), located in the 3rd quadrant, if $\tan t = \frac{8}{15}$?

$\sec^2 t = 1 + \tan^2 t = \frac{289}{225}$; $\sec t = \frac{-17}{15}$; $\cos t = \frac{-15}{17}$; $\sin t = \frac{-8}{17} \Rightarrow P(t) = \left(\frac{-15}{17}, \frac{-8}{17}\right)$

8. Find the exact Cartesian coordinates of the trigonometric point $P\left(-\frac{19\pi}{6}\right)$.

$P\left(\frac{-19\pi}{6}\right) = P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. Knowing that $\sin a = \frac{3}{5}$, $\cos a = \frac{4}{5}$, $\sin b = \frac{5}{13}$, $\cos b = \frac{12}{13}$, find the value of

a) $\sin(a + b)$ — $\sin a \cos b + \sin b \cos a = \frac{56}{65}$

b) $\cos(a - b)$ — $\cos a \cos b + \sin a \sin b = \frac{63}{65}$

c) $\tan(a + b)$ — $\frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$

10. Simplify $\frac{\sin 2\theta}{1 + \cos 2\theta}$. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2 \sin\theta \cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$

11. Simplify $\frac{\sin 4a \cos 2a - \sin 2a \cos 4a}{\cos 2a \cos a + \sin 2a \sin a}$. $\frac{\sin(4a - 2a)}{\cos(2a - a)} = \frac{\sin 2a}{\cos a} = \frac{2 \sin a \cos a}{\cos a} = 2 \sin a.$

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13. If $P\left(\frac{a+2}{5}, \frac{a+1}{5}\right)$ is a trigonometric point located in the 3rd quadrant, determine a.

$\left(\frac{a+2}{5}\right)^2 + \left(\frac{a+1}{5}\right)^2 = 1$; $2a^2 + 6a - 20 = 0$; $a = -5$ or $a = 2$.

Reject the solution $a = 2$ since the trigonometric point is located in the 3rd quadrant.

14. If $\sin t = \frac{15}{17}$ and $\frac{\pi}{2} \leq t \leq \pi$, find

a) $\cos t$ $\frac{-8}{17}$ **b)** $\tan t$ $\frac{-15}{8}$ **c)** $\cot t$ $\frac{-8}{15}$ **d)** $\sec t$ $\frac{-17}{8}$ **e)** $\csc t$ $\frac{17}{15}$

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19. Solve the following equations over $[0, 2\pi[$.

a) $2\sin^2 x - 1 = 0$ $S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

b) $(2\cos x + 1)(\sin x - 3) = 0$ $S = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

c) $25\sin^2 x - 9 = 0$ $S = \{0.64; 2.50; 3.78; 5.64\}$

d) $(5\cos x + 2)(2\sin x - 1) = 0$ $S = \left\{1.98; 4.30; \frac{\pi}{6}; \frac{5\pi}{6}\right\}$

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24. Prove the following identities.

a) $\frac{1-\sin t}{\cos t} = \frac{\cos t}{1+\sin t}$

$\frac{1-\sin t}{\cos t} = \frac{(1-\sin t)(1+\sin t)}{\cos t\,(1+\sin t)} = \frac{1-\sin^2 t}{\cos t\,(1+\sin t)} = \frac{\cos^2 t}{\cos t\,(1+\sin t)} = \frac{\cos t}{1+\sin t}$

b) $\frac{\sin 2t}{1+\cos 2t} = \tan t$

$\frac{\sin 2t}{1+\cos 2t} = \frac{2\sin t\cos t}{1+(2\cos^2 t-1)} = \frac{2\sin t\cos t}{2\cos^2 t} = \frac{\sin t}{\cos t} = \tan t$

c) $\cos^4 t - \sin^4 t = \cos 2t$

$\cos^4 t - \sin^4 t = (\cos^2 t + \sin^2 t)(\cos^2 t - \sin^2 t) = 1(\cos 2t) = \cos 2t$

d) $\frac{\sec t - 1}{\tan t} = \csc t - \cot t$

$\frac{\sec t - 1}{\tan t} = (\sec t - 1)\frac{\cos t}{\sin t} = \frac{1}{\sin t} - \frac{\cos t}{\sin t} = \csc t - \cot t$