Evaluation 1

1. Expand the following expressions using the appropriate identity.

a) $(3x^2 + 2x)^2$ ***$9x^4 + 12x^3 + 4x^2$***

b) $(2x^3 - 3y^2)^2$ ***$4x^6 - 12x^3y^2 + 9y^4$***

c) $(3a^2 + 2b)(3a^2 - 2b)$ ***$9a^4 - 4b^2$***

d) $(2x + 1)(2x - 1)(4x^2 + 1)$ ***$16x^4 - 1$***

2. A rectangular base prism has the volume $V(x) = 4x^3 + 6x^2 - 16x - 24$. The dimensions of the prism's base are $(2x + 3)$ and $(x - 2)$. Determine the height of this prism. ***$2x + 4$***

3. Factor the following polynomials.

a) $12a^3b^2 - 18a^2b^4$ ***$6a^2b^2(2a - 3b^2)$***

b) $(2x + 3)(x - 2) - (2x + 3)(2x + 1)$ ***$(2x + 3)(-x - 3)$***

c) $(2x + 1)^2 + (2x + 1)(x - 5)$ ***$(2x + 1)(3x - 4)$***

d) $6a^2 - 4ac + 9ab - 6bc$ ***$(2a + 3b)(3a - 2c)$***

e) $16x^4 - 25y^2$ ***$(4x^2 + 5y)(4x^2 - 5y)$***

4. Factor the following trinomials.

a) $x^2 + 2x - 15$ ***$(x - 3)(x + 5)$***

b) $2x^2 - x - 6$ ***$(2x + 3)(x - 2)$***

c) $9x^2 - 30xy + 25y^2$ ***$(3x - 5y)^2$***

d) $4x^4 + 16x^3 + 16x^2$ ***$4x^2(x + 2)^2$***

5. Factor the following expressions.

a) $16x^2 - (2x - 1)^2$ ***$(6x - 1)(2x + 1)$***

b) $x^4 - 18x^2 + 81$ ***$(x + 3)^2 \cdot (x - 3)^2$***

c) $x^4 - 81$ ***$(x + 3)(x - 3)(x^2 + 9)$***

d) $6x^3 - 4x^2 - 2x$ ***$2x(3x + 1)(x - 1)$***

6. Simplify the following rational expressions after indicating the restrictions on the variable.

a) $\dfrac{(x + 1)^2 - 16}{x^2 + 8x + 15}$ ***$\dfrac{x - 3}{x + 3}$, $x \neq -5$ and $x \neq -3$***

b) $\dfrac{2xy + 10x + 3y + 15}{y^2 - 25}$ ***$\dfrac{2x + 3}{y - 5}$, $y \neq -5$ and $y \neq 5$***

c) $\dfrac{4ab - 6a + 8b - 12}{4b - 6}$ ***$a + 2$, $b \neq \dfrac{3}{2}$***

d) $\dfrac{2x^3 + 4x^2 - 6x}{2x^2 + 6x}$ ***$x - 1$, $x \neq 0$; $x \neq -3$***

7. Perform the following operations given that variables satisfy the restrictions.

a) $\dfrac{x}{x - 3} + \dfrac{2x - 6}{x^2 - 6x + 9} =$ ***$\dfrac{x + 2}{x - 3}$***

b) $\dfrac{x^2 + 10x + 25}{2x^2 + 9x - 5} \times \dfrac{3}{3x + 15} =$ ***$\dfrac{1}{2x - 1}$***

c) $\dfrac{4x^2 - 12x + 9}{3x^2 - 5x} \div \dfrac{2x - 3}{3x - 5} =$ ***$\dfrac{2x - 3}{x}$***

d) $\dfrac{x^2}{2y - 2} \div \dfrac{x}{y^2 - 1} =$ ***$\dfrac{x(y + 1)}{2}$***

8. Solve the following equations.

a) $(2x - 1)(x - 3) = 0$ $S = \left\{\frac{1}{2}, 3\right\}$

b) $x^2 = x$ $S = \{0, 1\}$

c) $4x^2 - 9 = 0$ $S = \left\{-\frac{3}{2}, \frac{3}{2}\right\}$

d) $x^2 + 1 = 0$ $S = \emptyset$

e) $(x - 3)^2 - 4 = 0$ $S = \{1, 5\}$

f) $2(x - 1)^2 - 8 = 0$ $S = \{3, -1\}$

9. Solve the following equations.

a) $2x^2 - 9x - 5 = 0$ $S = \left\{-\frac{1}{2}, 5\right\}$

b) $4x^2 - 12x + 9 = 0$ $S = \left\{\frac{3}{2}\right\}$

c) $x^2 - 2x + 3 = 0$ $S = \emptyset$

d) $(x + 1)(x + 2) = 6$ $S = \{-4, 1\}$

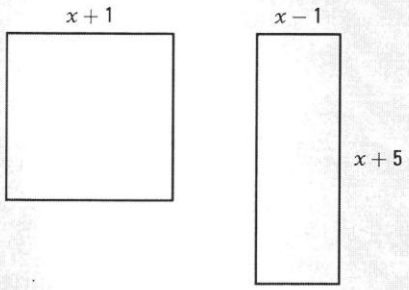
e) $(2x + 3)^2 = (x - 2)^2$ $S = \left\{-\frac{1}{3}, -5\right\}$

f) $2x^3 + 6x^2 + 4x = 0$ $S = \{0, -1, -2\}$

10. The square and the rectangle on the right have the same area. Determine the perimeter of the rectangle.

$(x + 1)^2 = (x - 1)(x + 5)$

$x = 3 \Rightarrow$ Perimeter of the rectangle = 20 units



11. Solve the following inequalities.

a) $x^2 \geqslant x$ $]-\infty, 0] \cup [1, +\infty[$

b) $x^2 \geqslant 9$ $]-\infty, -3] \cup [3, +\infty[$

c) $x^2 + 3x - 4 \geqslant 0$ $]-\infty, -4] \cup [1, +\infty[$

d) $-x^2 + 5x - 6 \geqslant 0$ $[2, 3]$

e) $x^2 - 6x + 9 \leqslant 0$ $\{3\}$

f) $x^2 - x + 1 \geqslant 0$ $\mathbb{R}$

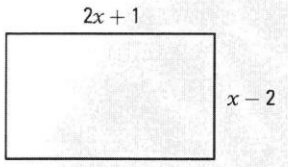
12. The polynomial $P(x) = -x^2 + 6x + 4$ enables you to calculate the price $P(x)$ of a share x months after its purchase.

a) What is the share's purchase value? $4

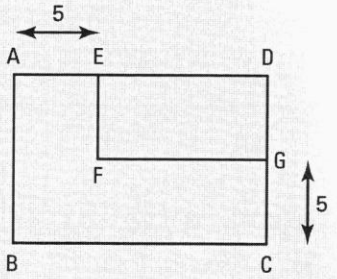
b) If the value of the share is greater than \$9, in what interval must the number of elapsed months since the share's purchase be? $]1, 5[$

13. Determine the interval in which the variable x must be located for the area of the rectangle on the right to be greater than 18 u^2?

$x \in]4, +\infty[$



14. Consider the rectangles ABCD and DEFG represented on the right. Knowing that $m\overline{AE} = m\overline{CG} = 5$ units and that the polynomial $6x^2 + 5x - 25$ represents the area of rectangle ABCD, determine the polynomial which represents the area of rectangle DEFG.



$6x^2 + 5x - 25 = (3x - 5)(2x + 5)$

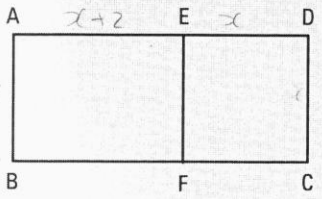
Dimensions of DEFG: $(3x - 10)$ and $2x$

Area of rectangle DEFG = $6x^2 - 20x$

15. The polynomial $2x^3 + 11x^2 + 15x$ represents the volume of a rectangular base prism. The dimensions of the prism's base are x and $(x + 3)$.

What binomial represents the height of the prism? ***$2x + 5$***

16. Consider the rectangle ABFE and the square CDEF on the right. Segment AE measures 2 units more than segment ED.

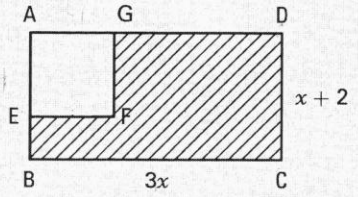


What is the numerical value of the area of rectangle ABFE if the area of rectangle ABCD is equal to 40 cm^2?

• $m\overline{ED} = x$; $m\overline{AE} = x + 2$; Area of ABCD = $x(2x + 2) = 2x^2 + 2x$

$2x^2 + 2x = 40 \Rightarrow x = 4 \Rightarrow$ Area of ABFE = 24 cm^2

17. The dimensions of the rectangle ABCD on the right are $3x$ and $(x + 2)$. The quadrilateral AEFG is a square with an area of 16 cm^2. The area of the shaded region is equal to $(2x^2 + 6x)$ cm^2. What is the numerical value of the area of rectangle ABCD?



$3x(x + 2) = 2x^2 + 6x + 16 \Rightarrow x = 4$

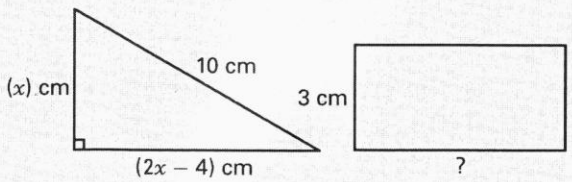
Area of rectangle ABCD = $12 \times 6 = 72$ cm^2

18. The polynomial $h(t) = -t^2 + 6t + 6$ enables you to calculate the height $h(t)$, in metres, of an object t seconds after it is launched. Between what instants after its launch does the object reach a height greater than 14 m?

Between the instants $t = 2s$ and $t = 4s$.

19. The given right triangle and rectangle have the same area.

What is the numerical value of the rectangle's length?



$x = 6$; length = 8 cm