Science Math Review

1. In the following algebraic expression, the denominator is not equal to zero.

$$\frac{3x^2+13x+14}{x+2}$$

What binomial is equivalent to this expression?

2. The rule of the function is: $f(x)=2x^2-3x-5$. What are the zeros of this function?

3. The volume of a right prism is $(2x^3+3x^2-11x-6)$ and its height is $(x-2)$ cm. What two binomial algebraic expressions could represent the length and width of the base of this prism?

4. Over which interval is the function $f(x)=-2x+26$ positive?

5. If b > 1 and c ≠ 1, what is the simplified result of the following operation?

$$\frac{b^3}{2(c-1)} \div \frac{1}{b^2c-b^2}$$

6. Solve for x: $x(2x-1)=15$

7. Simplify:

$$\frac{x-3}{x^2+2x-15}-\frac{x^2+3x-18}{2x^2+13x+6}$$

8. Simplify:

$$\frac{10}{3x-3}\div\frac{3x-1}{x^2-1}\times\frac{9x^2-6x+1}{2x+2}$$

7. $\frac{-x^2+16}{(x+5)(2x+1)}$ 8. $\frac{5(3x-1)}{3}$

Answers

1. $3x+7$
2. $x=5/2$ or $x=-1$
3. $(x+3)(2x+1)$
4. $]-\infty,13]$
5. $b^5/2$
6. $x=-2.5$ $x=3$

[redacted]

1. ⓐ $\frac{3x^2+13x+14}{x+2}$

$$\begin{array}{r|l} & 3x+7 \\ \hline x+2 & 3x^2+13x+14 \\ & -(3x^2+6x) \\ \hline & \quad 7x+14 \\ & \quad -(7x+14) \\ \hline & \qquad 0 \end{array}$$

Answer: $3x+7$

ⓑ $\frac{3x^2+13x+14}{x+2} \Rightarrow$

$m \times n = 42$
$m+n = 13$
$m,n = 6,7$
$3x^2+6x+7x+14$
$3x(x+2)+7(x+2)$
$(x+2)(3x+7)$

$\frac{(3x+7)\cancel{(x+2)}}{\cancel{(x+2)}}$

Answer: $3x+7$

2. $0 = 2x^2-3x-5$

ⓐ $m \times n = -10$
$m+n = -3$
$m,n = -5,2$

$0 = 2x^2+2x-5x-5$
$0 = 2x(x+1)-5(x+1)$
$0 = (x+1)(2x-5)$
$x+1=0 \qquad 2x-5=0$
$x=-1 \qquad 2x=5$
$x=5/2$
$\therefore x = \{-1, 5/2\}$

ⓑ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{3 \pm \sqrt{9-4(2)(-5)}}{4}$

$x = \frac{3 \pm \sqrt{9+40}}{4}$

$x = \frac{3 \pm \sqrt{49}}{4}$

$x = \frac{3 \pm 7}{4}$

$x = \frac{10}{4}$ or $x = \frac{-4}{4}$

$x = \{-1, 5/2\}$

3. $\quad x-2 \,\overline{)\, 2x^3+3x^2-11x-6}$ with quotient $2x^2+7x+3$

$-(2x^3-4x^2)$

$7x^2-11x$

$-(7x^2-14x)$

$3x-6$

$-(3x-6)$

0

$\therefore \quad L \times W = 2x^2+7x+3$

$m \times n = 6 \quad m+n=7$

$m,n = 6,1$

$2x^2+6x+x+3$

$2x(x+3)+1(x+3)$

$(x+3)(2x+1)$

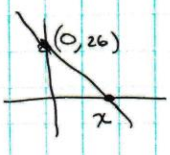
$\therefore$ Length & width are $(x+3)$ and $(2x+1)$

4. $f(x) = -2x+26$

$0 = -2x+26$

$-26 = -2x$

$13 = x$



positive $(-\infty, x]$

Positive : $(-\infty, 13]$

5. $\dfrac{b^3}{2(c-1)} \div \dfrac{1}{b^2c-b^2} = \dfrac{b^3}{2(c-1)} \div \dfrac{1}{b^2(c-1)}$

$= \dfrac{b^3}{2\cancel{(c-1)}} \times \dfrac{b^2\cancel{(c-1)}}{1}$

$= \dfrac{b^5}{2}$

6. $x(2x-1)=15$

$2x^2-x=15$

$2x^2-x-15=0$

$m \times n = -30 \quad m+n=-1$

$-6, 5$

$2x^2-6x+5x-15=0$

$2x(x-3)+5(x-3)=0$

$(x-3)(2x+5)=0$

$x-3=0 \quad 2x+5=0$

$x=3 \quad 2x=-5$

$x=-5/2$

$x = \dfrac{1 \pm \sqrt{1-4(2)(-15)}}{4} = \dfrac{1 \pm \sqrt{121}}{4}$

$= \dfrac{1 \pm 11}{4}$

$x = \dfrac{12}{4} \qquad x = \dfrac{-10}{4}$

$x = 3 \qquad x = -5/2$

$x = \left\{-\dfrac{5}{2}, 3\right\}$

7. $$\frac{x-3}{x^2+2x-15} - \frac{x^2+3x-18}{2x^2+13x+6} = \frac{x-3}{(x+5)(x-3)} - \frac{(x+6)(x-3)}{(2x+1)(x+6)}$$

$$x \neq \{-6, -5, ^{-}1/2, 3\}$$

$$= \frac{1}{x+5} - \frac{x-3}{2x+1}$$

$$= \left(\frac{2x+1}{2x+1}\right)\left(\frac{1}{x+5}\right) - \left(\frac{x-3}{2x+1}\right)\left(\frac{x+5}{x+5}\right)$$

$$= \frac{2x+1}{(2x+1)(x+5)} - \frac{(x-3)(x+5)}{(2x+1)(x+5)}$$

$$= \frac{2x+1-(x^2+2x-15)}{(2x+1)(x+5)}$$

$$= \frac{-x^2+16}{(2x+1)(x+5)}$$

$$= \frac{16-x^2}{(2x+1)(x+5)}$$

OR

$$= \frac{(4+x)(4-x)}{(2x+1)(x+5)}$$

8. $$\frac{10}{3x-3} \div \frac{3x-1}{x^2-1} \times \frac{9x^2-6x+1}{2x+2}$$

$$\frac{10}{3(x-1)} \div \frac{3x-1}{(x+1)(x-1)} \times \frac{(3x-1)(3x-1)}{2(x+1)} \quad , \quad x \neq \{-1, 1\}$$

$$\frac{\overset{5}{\cancel{10}}}{3\cancel{(x-1)}} \times \frac{\cancel{(x+1)}\cancel{(x-1)}}{\cancel{3x-1}} \times \frac{\cancel{(3x-1)}(3x-1)}{\cancel{2}\cancel{(x+1)}} \quad , \quad x \neq \{-1, 1/3, 1\}$$

$$\frac{5(3x-1)}{3}$$

Reasoning Practice

1. The rule of quadratic function g is of the form $g(x) = a(x - h)^2 + k$.

This function g is positive over the interval $[-5, 11]$.

Its range is $]-\infty, 128]$.

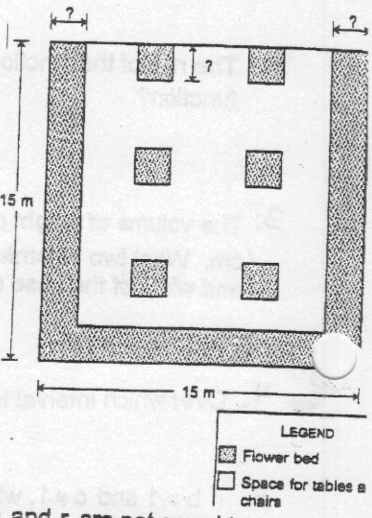
In the rule of function g, what is the numerical value of a?

2.

A restaurant owner wants to set up an outdoor terrace.

- The terrace will be square, with each of its sides measuring 15 meters.
- The terrace will include 7 flower beds; 6 will be square and 1 will be U-shaped, as shown.
- Tables and chairs will be set up in the rest of the unoccupied space.
- The 6 square flowerbeds are congruent.
- The width of the U-shaped flowerbed must be equal to the measure of one side of a square flowerbed.
- The total area of the flowerbeds must be at most 62.5 m² in order to limit the amount of maintenance they will require.

LAYOUT PLAN FOR THE TERR



What is the maximum possible width of the flowerbeds?

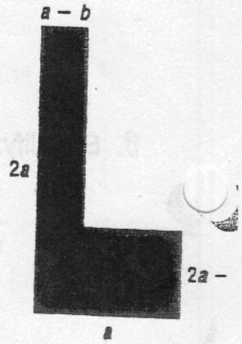
3. The rule of function g is $g(x) = px^2 + rx - 36$, where p and r are not equal to zero.

Function g is positive over the intervals $]-\infty, -6] \cup [10, +\infty[$.

What is the value of p?

4. **A sheet of plywood is twice as long as its width. A carpenter cuts out a rectangle of which the length is also double that of the width. The carpenter is then left with an L-shaped piece.**

Represent the area of this piece as a product of factors.



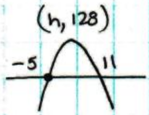
1. $a = -2$

2. 1.25 m

4. $2(a+b)(a-b)$

3. $p = a = \frac{3}{5}$

1. $g(x) = a(x-h)^2 + k$



$h = \frac{-5+11}{2} = \frac{6}{2} = 3$

$\therefore (h,k) = (3,128)$

$g(x) = a(x-3)^2 + 128$

$0 = a(11-3)^2 + 128$

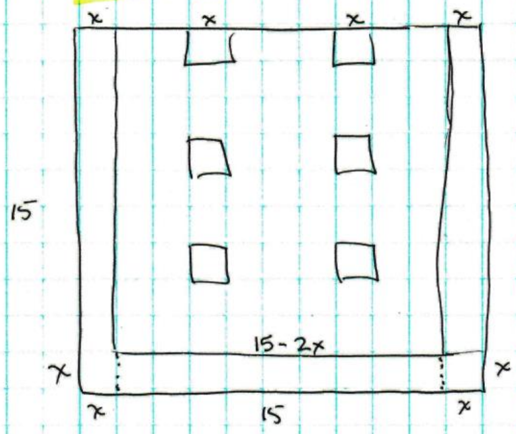
$0 = a(8)^2 + 128$

$0 = 64a + 128$

$-128 = 64a$

$-2 = a$

2.



Flower beds: $6 \times x^2$

$2 \times 15x$

$1 \times x(15-2x)$

Total: $6x^2 + 30x + 15x - 2x^2$

$4x^2 + 45x$

$4x^2 + 45x = 62.5$

$4x^2 + 45x - 62.5 = 0$

$m \times n = {}^{-}250 \quad m+n = 45$

$50, -5$

$4x^2 + 50x - 5x - 62.5 = 0$

$4x(x+12.5) - 5(x+12.5) = 0$

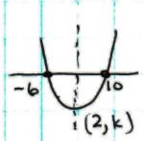
$(4x-5)(x+12.5) = 0$

$x = \frac{5}{4} \qquad x = -12.5$

Answer: $\frac{5}{4}$m or 1.25m

$x = \frac{5}{4}$ or 1.25

3. $g(x) = px^2 + rx - 36$



$h = \frac{-6+10}{2} = 2$

y-int = $(0, {}^{-}36)$

$k < {}^{-}36$

$g(x) = a(x+6)(x-10)$

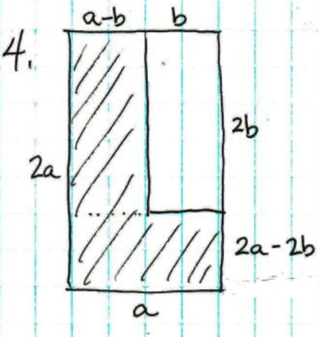
${}^{-}36 = a(0+6)(0-10)$

${}^{-}36 = a(-60)$

$+\frac{3}{5} = {}^{-}36/{-60} = a$

$\therefore p = \frac{3}{5}$

4.



$a(2a - 2b) + (a-b)(2b)$

$a \cdot 2(a-b) + (a-b)(2b)$

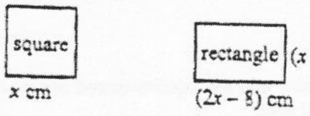
$2(a-b)(a+b)$

PROBLEM-SOLVING REVIEW for the Mid-Term Exam

1. The function h(t) represents the height of an object (meters) as a function of time (seconds). For how long is the projectile more than 31 meters off the ground?

use Q.F. $h(t) = -.75t^2 + 10t + 15$

2. The following square and rectangle are equivalent.

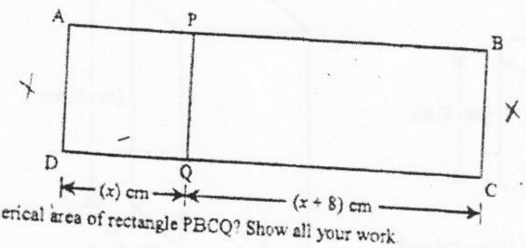


What are the actual dimensions of the rectangle?

Show all your work.

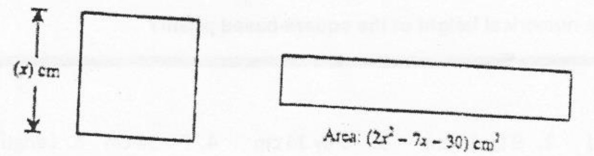
3. In the figure below, line segment PQ divides rectangle ABCD into the following two quadrilaterals: square APQD and rectangle PBCQ.

The area of rectangle ABCD is 120 cm². In addition, m $\overline{DQ}$ = (x) cm and m $\overline{QC}$ = (x + 8) cm.



What is the numerical area of rectangle PBCQ? Show all your work.

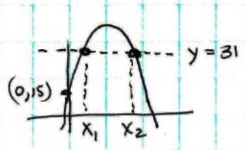
4. The square and the rectangle shown below are equivalent figures. (have same area) Each side of the square measures X cm. The area of the rectangle is $(2x^2 - 7x - 30)$ cm².



What is the perimeter of the rectangle?

Your final answer must be a number. Show all your work.

1. $h(t) = -0.75t^2 + 10t + 15$



let $y = 31$

$31 = -0.75t^2 + 10t + 15$
$0 = -0.75t^2 + 10t - 16$

$$t = \frac{-10 \pm \sqrt{100 - 4(-0.75)(-16)}}{-1.5}$$

$$t = \frac{-10 \pm \sqrt{52}}{-1.5}$$

$t_1 \approx 1.86$ or $t_2 \approx 11.47$

$]1.86, 11.47[$

length of time > 31 m
$11.47 - 1.86$
≈ 9.61 seconds

2.

(x^2) cm² $(2x^2 - 14x + 24)$ cm²

$x^2 = 2x^2 - 14x + 24$
$0 = x^2 - 14x + 24$
$0 = (x-12)(x-2)$
~~$x = 2$~~ or $x = 12$

$x = 12$

$2x - 8 = 16$ cm
$x - 3 = 9$ cm

3. $x^2 + x(x+8) = 120$

$x^2 + x^2 + 8x = 120$

$2x^2 + 8x - 120 = 0$

$x^2 + 4x - 60 = 0$

$(x+10)(x-6) = 0$

$x = -10$ X

$\boxed{x = 6}$

$x = 6$

$x + 8 = 14$

Area PBCQ $= 6 \times 14$

$= 84\ \text{cm}^2$

4. $x^2 = 2x^2 - 7x - 30$

$0 = x^2 - 7x - 30$

$0 = (x-10)(x+3)$

$x = 10$, $x = -3$ x

$x = 10$

$2x^2 - 7x - 30$

$(2x+5)(x-6) = L \times W$

let $x = 10$

$2x + 5 = 25$ cm

$x - 6 = 4$ cm

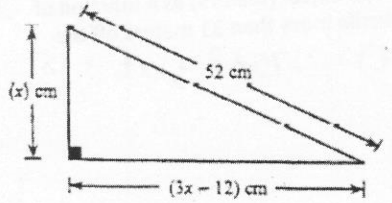
$P = 2(25) + 2(4)$

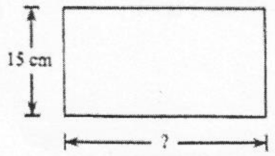
$= 58$ cm

5. The right triangle and the rectangle given below are equivalent.

The hypotenuse of the triangle measures 52 cm. The sides of the right angle of the triangle measure (x) cm and $(3x-12)$ cm respectively.

The height of the rectangle is 15 cm.



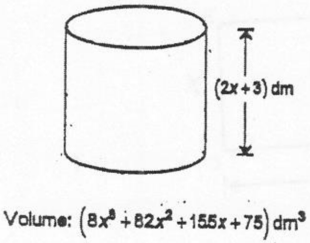


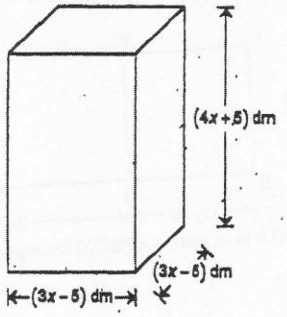
What is the numerical length of the base of the rectangle? Show all your work.

6. The right cylinder and the square-based prism shown below have bases of equal area.

The height of the cylinder is represented by the binomial $(2x+3)$dm. It's volume is represented by the polynomial $8x^3+82x^2+155x+75$ dm^3.

The lengths of the edges of the base of the prism are represented by the binomial $(3x-5)$ dm and its height is represented by the binomial $(4x+5)$dm.





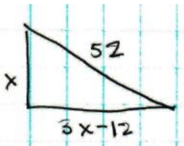
The volume of both prisms is calculated by $V = A_{base} \times height$

What is the numerical height of the square-based prism?

ANSWERS:

1.]1.86 sec, 11.47 sec[**2.** 9 by 16 cm **3.** 6 by 14 cm **4.** P = 58 cm **5.** Length = 32 cm **6.** H = 57 dm

5.



① $x^2 + (3x - 12)^2 = 52^2$

$x^2 + 9x^2 - 72x + 144 = 2704$

$10x^2 - 72x - 2560 = 0$

$$x = \frac{72 \pm \sqrt{72^2 - 4(10)(-2560)}}{20}$$

$$x = \frac{72 \pm \sqrt{107\,584}}{20}$$

$$x = \frac{72 \pm 328}{20}$$

$x = 20$ or $\cancel{x = -128}$

② $x = 20$

$3x - 12 = 48$

$$\text{Area} = \frac{48 \times 20}{2}$$

$= 480\ \text{cm}^2$

③ $L \times W = 15 \times ?$

$480 = 15(?)$

$32 = ?$

32 cm

6. Cylinder $V = A_b \times h \Rightarrow 8x^3 + 82x^2 + 155x + 75 = A_b \times (2x+3)$

$$\begin{array}{r} 4x^2 + 35x + 25 \\ 2x+3 \overline{)\, 8x^3 + 82x^2 + 155x + 75} \\ \underline{-(8x^3 + 12x^2)} \\ 70x^2 + 155x \\ \underline{-(70x^2 + 105x)} \\ 50x + 75 \\ \underline{-(50x + 75)} \\ 0 \end{array}$$

$A_b = 4x^2 + 35x + 25$

Prism: $A_b = (3x-5)(3x-5) = 9x^2 - 30x + 25$

Equal Bases: $9x^2 - 30x + 25 = 4x^2 + 35x + \cancel{25}$

$5x^2 - 65x = 0$

$5x(x - 13) = 0$

$\cancel{x = 0}$ or $x = 13$

Height of Prism $= 4x + 5$

$= 52 + 5$

$= 57$ cm

Math 408 – Mid-Term Exam Review (SHORT-ANSWER QUESTIONS)

1. Given the following rule: f(x) = -0.016x (x-100), find the interval of decrease.

2. Find the rule corresponding to the given table of values.

x	0	1	2	3	4	5	6
y	0	-4	-4	0	8	20	36

3. What is the equation of the axis of symmetry of the given parabola: $f(x) = 2(x + 3)^2 - 4$

4. Solve the following equations: a) $2(x + 3)(x - 4) = 0$ b) $-2x^2 + 3x = 8$ c) $2(x - 4)^2 - 8 = 0$

5. What is the rule of a quadratic function which is positive over the interval $]-\infty, 2] \cup [5, +\infty[$ and its y-intercept is 5?

6. What is the axis of symmetry of a quadratic function with zeros -3 and 5?

7. Which perfect square comes after 121?

8. Determine the rule, in general form, of a quadratic function with a maximum of 8 and going through A(1, 3) and B(5, 3).

9. What is the range of a parabola that has only one zero which is 4, and a y-intercept of -4?

10. Indicate if this is a perfect square trinomial: $9x^2 - 24xy + 16y^2$.

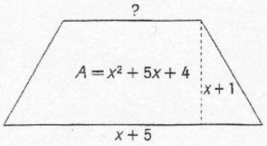
11. Factor the following completely: $x^4 - 1$

12. Subtract: $\frac{x+1}{x^2+2x+1} - \frac{x+3}{x^2+4x+3}$

13. Simplify: $\frac{2y^2}{y+1} \div \frac{6y}{xy+x}$

14. Simplify using positive exponents only: $\left(\frac{2x^3y^{-2}}{y^{-1}}\right)^2 \left(\frac{\sqrt{x^{-2}}}{x^{-1}}\right)^2$

15. Find the missing polynomial:



16. Divide: $\frac{x^3+27}{x+3}$

17. Determine f(6) if $f(x) = -3(x - 3)^2 + 4$. Is point (-3, 112) defined by this function?

1. $f(x) = -0.016x(x-100)$ zeros are $\{0, 100\}$ $h = \frac{100}{2} = 50$

a is negative

Decreasing $[50, +\infty)$

2. zeros $\{0, 3\}$

$f(x) = a(x-0)(x-3)$

$20 = a(5)(5-3)$

$20 = a(5)(2)$

$20 = 10a$

$2 = a$

$\therefore f(x) = 2x(x-3)$ or $f(x) = 2x^2 - 6x$

3. Axis of symmetry : $x = -3$

4. a) $2(x+3)(x-4) = 0$

$\div 2 \qquad \div 2$

$(x+3)(x-4) = 0$

$x+3=0$ or $x-4=0$

$x=-3 \qquad x=4$

$x = \{-3, 4\}$

b) $-2x^2+3x=8$

$-2x^2+3x-8=0$

$m \times n = 16 \quad m+n=3$

$x = \frac{-3 \pm \sqrt{9-4(-2)(-8)}}{-4}$

$x = \frac{-3 \pm \sqrt{-55}}{-4}$

No solution

c) $2(x-4)^2-8=0$

$2(x-4)^2 = 8$

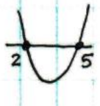
$(x-4)^2 = 4$

$x-4 = \pm 2$

$x-4=2$ or $x-4=-2$

$x=6 \qquad x=2$

$x = \{2, 6\}$

5. 

$P(0,5)$

$f(x) = a(x-2)(x-5)$

$5 = a(0-2)(0-5)$

$5 = a(-2)(-5)$

$5 = 10a$

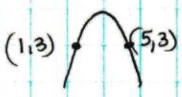
$\frac{1}{2} = a$

$\therefore f(x) = \frac{1}{2}(x-2)(x-5)$

6. $h = -\frac{3+5}{2} = 1$ Axis of symmetry : $x = 1$

7. $121 = 11^2$ $\therefore$ next perfect square $= 12^2 = 144$

8 $k = 8$ $A(1,3)$ $B(5,3)$



$h = \frac{1+5}{2} = 3$

$\therefore V(3,8)$

$f(x) = a(x-3)^2 + 8$

$3 = a(5-3)^2 + 8$

$3 = a(2)^2 + 8$

$-5 = 4a$

$-1.25 = -5/4 = a$

$\therefore f(x) = -5/4(x-3)^2 + 8$

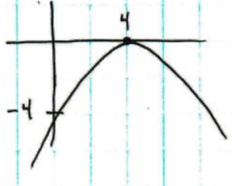
$f(x) = -5/4(x^2 - 6x + 9) + 8$

$f(x) = -\frac{5}{4}x^2 + \frac{15}{2}x - \frac{45}{4} + 8$

$f(x) = -\frac{5}{4}x^2 + \frac{15}{2}x - \frac{45}{4} + \frac{32}{4}$

$f(x) = -5/4x^2 + \frac{15}{2}x - \frac{13}{4}$ OR $f(x) = -1.25x^2 + 7.5x - 3.25$

9.



$V(4,0)$

Ran $= (-\infty, 0]$

10. $9x^2 - 24xy + 16y^2$ $\sqrt{9x^2} = 3x$ $\sqrt{16y^2} = 4y$

$2 \times 3x \times 4y = 24xy$

Since the trinomial follows the pattern

$(a+b)^2 = a^2 + 2ab + b^2$

it is a perfect square trinomial

OR

$9x^2 - 24xy + 16y^2$

$m \times n = 144$

$m + n = -24$

$m, n = -12, -12$

$9x^2 - 12xy - 12xy + 16y^2$

$3x(3x - 4y) - 4y(3x - 4y)$

$(3x - 4y)(3x - 4y)$

$(3x - 4y)^2$

11. $x^4 - 1 = (x^2+1)(x^2-1)$

$$= (x^2+1)(x+1)(x-1)$$

12. $\frac{x+1}{x^2+2x+1} - \frac{x+3}{x^2+4x+3} = \frac{\cancel{x+1}}{(x+1)\cancel{(x+1)}} - \frac{\cancel{x+3}}{(x+1)\cancel{(x+3)}} \qquad x \neq \{-3, -1\}$

$$= \frac{1}{x+1} - \frac{1}{x+1}$$

$$= 0 \, , \; x \neq \{-3, -1\}$$

13. $\frac{2y^2}{y+1} \div \frac{6y}{xy+x} = \frac{2y^2}{y+1} \div \frac{6y}{x(y+1)} \, , \quad x \neq 0 \, , \; y \neq -1$

$$= \frac{\cancel{2}y^{\cancel{2}}}{\cancel{y+1}} \times \frac{x\cancel{(y+1)}}{{}_{3}\cancel{6}\cancel{y}} \, , \quad x \neq 0, \; y \neq \{-1, 0\}$$

$$= \frac{xy}{3} \, , \quad x \neq 0, \; y \neq \{-1, 0\}$$

14. $\left(\frac{2x^3y^{-2}}{y^{-1}}\right)^2 \left(\frac{\sqrt{x^{-2}}}{x^{-1}}\right)^2 = \left(2x^3y^{-1}\right)^2 \left(\frac{(x^{-2})^{\frac{1}{2}}}{x^{-1}}\right)^2$

$$= \left(4x^6y^{-2}\right)\left(\frac{x^{-1}}{x^{-1}}\right)^2$$

$$= 4x^6y^{-2}\,(1)^2$$

$$= 4x^6y^{-2}$$

$$= \frac{4x^6}{y^2}$$

15. $A = x^2 + 5x + 4 \qquad A = \frac{(B+b)h}{2}$

$$x^2 + 5x + 4 = \frac{(x+5+b)(x+1)}{2}$$

$$2(x^2+5x+4) = (x+5+b)(x+1)$$

$$2(x+4)(x+1) = (x+5+b)(x+1)$$

$$2(x+4) = x+5+b \, , \quad x \neq -1$$

$$2x + 8 = x+5+b$$

$$x + 3 = b$$

16. $\frac{x^3+27}{x+3}$

$x^2 - 3x + 9$

$x+3 \overline{) x^3 + 0x^2 + 0x + 27}$

$-(x^3 + 3x^2)$

$-3x^2 + 0x$

$-(-3x^2 - 9x)$

$9x + 27$

$-(9x + 27)$

0

Answer: $x^2 - 3x + 9$

17. $f(6) = -3(6-3)^2 + 4$

$= -3(3)^2 + 4$

$= -3(9) + 4$

$= -27 + 4$

$= -23$

$f(-3) = -3(-3-3)^2 + 4$

$= -3(-6)^2 + 4$

$= -3(36) + 4$

$= -108 + 4$

$= -104$ $\therefore (-3, -104)$

No, $(-3, 112)$ is not defined by this function

OR

$f(x) = -3(x-3)^2 + 4$

the maximum is 4; therefore $(-3, 112)$ cannot be on the curve.