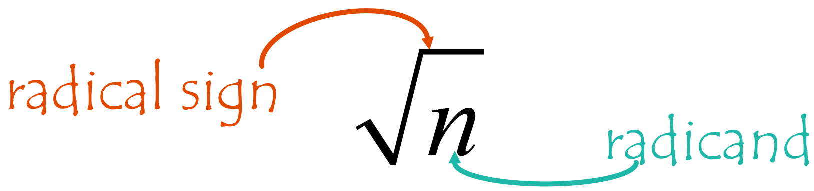
Radicals

Radicals are expressions that involve a root sign.

$\sqrt{n}$ is called a radical.



Addition and Subtraction

Adding and subtracting radicals is like algebra - they have to be "like terms" - the radicals must match.

Examples: $6\sqrt{7} - 4\sqrt{7} =$

$5\sqrt{2} + 3\sqrt{2} =$

$\sqrt{3} + 2\sqrt{5} + 10\sqrt{5} =$

Multiplication and Division

Properties: 1) $\sqrt{m} \times \sqrt{n} \Leftrightarrow \sqrt{m \times n}$ 2) $\frac{\sqrt{m}}{\sqrt{n}} \Leftrightarrow \sqrt{\frac{m}{n}}$

Example: $4\sqrt{2} \times 3\sqrt{6}$

Multiply/divide the coefficients and multiply/divide the radicands. Like terms are not necessary.

$$4\sqrt{2} \times 3\sqrt{6} =$$

$$=$$

Examples:

$$5\sqrt{20} \div 3\sqrt{10} =$$

$$=$$

$$2\sqrt{3} \times 6\sqrt{8} \div 4\sqrt{12} =$$

Simplifying Radicals

Many radicals can be simplified.

Example: $\sqrt{27}$

1. Break down the radicand into two factors - one of which is a perfect square.

   $\sqrt{27}$ = ____________

2. Apply the multiplication property of radicals to create a coefficient times a radical.

   ____

Example: $3\sqrt{72}$

Example: $4\sqrt{48}+2\sqrt{12}$

Example: $5\sqrt{98}+2\sqrt{200}$

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Try these!

1. $3\sqrt{5}\times\left(-2\sqrt{3}\right)$

2. $2\sqrt{3}\left(5\sqrt{3}+\sqrt{5}\right)$

3. $\left(4\sqrt{5}+3\right)\left(4\sqrt{5}-3\right)$

4. $12\sqrt{30}\div4\sqrt{5}$

5. $2\sqrt{75}-2\sqrt{108}+5\sqrt{75}-\sqrt{108}+3\sqrt{12}$

Simplify

1. $3\sqrt{5} \times -2\sqrt{3} = -6\sqrt{15}$

2. $2\sqrt{3}\left(5\sqrt{3} + \sqrt{5}\right) = 2\sqrt{3}\cdot 5\sqrt{3} + 2\sqrt{3}\cdot\sqrt{5}$
$= 10\sqrt{9} + 2\sqrt{15}$
$= 10\cdot 3 + 2\sqrt{15}$
$= 30 + 2\sqrt{15}$

3. $\left(4\sqrt{5} + 3\right)\left(4\sqrt{5} - 3\right) = 4\sqrt{5}\cdot 4\sqrt{5} + 4\sqrt{5}\cdot(-3) + 3\cdot 4\sqrt{5} + 3\cdot(-3)$
$= 16\sqrt{25} - 12\sqrt{5} + 12\sqrt{5} - 9$
$= 16(5) - 9$
$= 80 - 9$
$= 71$

4. $2\sqrt{75} - 2\sqrt{108} + 5\sqrt{75} - \sqrt{108} + 3\sqrt{12}$
$= 7\sqrt{75} - 3\sqrt{108} + 3\sqrt{12}$
$= 7\sqrt{25\cdot 3} - 3\sqrt{36\cdot 3} + 3\sqrt{4\cdot 3} = 35\sqrt{3} - 18\sqrt{3} + 6\sqrt{3}$
$= 23\sqrt{3}$

5. $12\sqrt{30} \div 4\sqrt{5} = \dfrac{12\sqrt{30}}{4\sqrt{5}}$
$= 3\sqrt{\dfrac{30}{5}}$
$= 3\sqrt{6}$

Rationalising the Denominator

We do not write expressions with a radical in the denominator.

Example: $\frac{6}{\sqrt{3}}$

We get rid of the radical in the denominator by a process called rationalising.

Multiply by a unit fraction that will square the denominator.

$$\frac{6}{\sqrt{3}}$$

Rationalise the denominators:

1. $\dfrac{4}{\sqrt{13}}$

2. $\dfrac{5\sqrt{3}}{3\sqrt{2}}$

3. $\dfrac{5\sqrt{6}}{12\sqrt{7}}$

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Example: $$\frac{5}{\sqrt{3}+\sqrt{6}}$$

When more than 1 term is in the denominator, we must use conjugates to create a difference of squares.

Example: $$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{2}-\sqrt{3}}$$

Rationalise the denominators:

1. $\dfrac{3}{2\sqrt{5}+\sqrt{6}}$

2. $\dfrac{4\sqrt{10}+2\sqrt{3}}{3\sqrt{2}-\sqrt{15}}$

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