

$$1. a) \cot 30^\circ = \frac{\sqrt{3/2}}{1/2} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

$$b) \sec \frac{37\pi}{6} = \sec \left(\frac{\pi}{6} \right) = \frac{2}{\sqrt{3}} \Rightarrow \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$c) \frac{\tan 45^\circ}{\csc 60^\circ} = \frac{\left(\frac{\sqrt{2/2}/\sqrt{2/2}}{2/\sqrt{3}} \right)}{2/\sqrt{3}} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$d) \tan \left(\frac{-33\pi}{4} \right) = \tan \frac{-\pi}{4} = \tan \frac{7\pi}{4} = \left(\frac{-\sqrt{2/2}}{\sqrt{2/2}} \right) = -1$$

$$e) \sin \frac{67\pi}{3} = \sin \pi/3 = \sqrt{3}/2$$

$$f) \csc \left(-\frac{113\pi}{3} \right) = \csc \frac{\pi}{3} = \frac{2}{\sqrt{3}} \Rightarrow \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$2. a) P(-1200^\circ) \Rightarrow \text{Quad } 3$$

$$b) -\frac{73\pi}{11} \Rightarrow 3$$

$$c) \frac{93\pi}{17} \Rightarrow 3$$

$$d) 492^\circ \Rightarrow 2$$

$$e) P\left(\frac{28\pi}{5}\right) \Rightarrow 4$$

$$f) 2.5 \text{ Rad} \Rightarrow 2$$

$$3. a) \sec \theta = -\frac{7}{5}$$

$$b) \cot \theta = \frac{-5/7}{2\sqrt{6}/7}$$

$$c) \cos \theta = -\frac{5}{7}$$

$$-5/7 \times \frac{7}{2\sqrt{6}}$$

$$-5/2\sqrt{6}$$

$$\frac{-5}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{-5\sqrt{6}}{12}$$

4. a) 120°

b) $\frac{7\pi}{6}$

c) $\frac{3\pi}{4}$

d) $\frac{\pi}{2}$

e) $\frac{\pi}{3}$

f) 207°

5. $\csc x = 2.366$

a) $\sin x = \frac{1}{2.366} \approx 0.423$

$x = \sin^{-1}(0.423)$
 $x = 0.436 \text{ Rad}$

b) $\cot x = \frac{1}{\tan x}$

$= \frac{1}{\tan 0.436}$
 $= \frac{1}{0.466}$
 ≈ 2.14

6. $L = \theta r$

$L = 30$

$r = \frac{L}{\theta}$

$\theta = \frac{3\pi}{8}$

$r = \frac{30}{\frac{3\pi}{8}}$

$r = 25.46 \text{ cm}$

$A = \pi r^2 = \pi (25.46)^2$
 $= 2037.18 \text{ cm}^2$

7. a) $\sec 40^\circ = \frac{x}{3}$

$x = 3 \sec 40^\circ$

$x = \frac{3}{\cos 40^\circ}$

$x = 3.92$

b) $\frac{\cos x}{\sin x} = 2.5$

$\frac{\sin x}{\cos x} = \frac{1}{2.5}$

$\tan x = 0.4$

$x = \tan^{-1} 0.4$

$x = 0.38 \text{ Rad}$

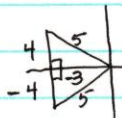
c) $\csc 51^\circ = x$

$\frac{1}{\sin 51^\circ} = x$

$\frac{1}{0.777} = x$

$1.29 = x$

8. $\cos \theta = -\frac{3}{5}$ $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$ (quadrants 2 & 3)



① $\sin \theta = \frac{4}{5}$

② $\sin \theta = -\frac{4}{5}$

$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{4/5}{-3/5} = \frac{4}{5} \times -\frac{5}{3}$

$= -\frac{4}{3}$

or

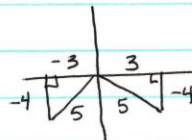
$= \frac{-4/5}{-3/5} = \frac{4}{3}$

9. $P(\theta) = \left(\frac{m}{n}, -\frac{4}{5}\right)$ $\therefore \sin \theta = -\frac{4}{5} \rightarrow$ in quadrant 3 or 4

$m = 3$ or -3

$n = 5$

$\cos \theta = \frac{m}{n} = -\frac{3}{5}$ or $\frac{3}{5}$



$\cot \theta = \frac{\cos \theta}{\sin \theta}$

$= \frac{-3/5}{-4/5} \Rightarrow \frac{-3 \times 5}{5 \times -4} = \frac{-3}{-4} = \frac{3}{4}$

or $\frac{3/5}{-4/5} \Rightarrow \frac{-3}{4}$

10. a) $P\left(\frac{\pi}{5}\right) = \left(\cos \frac{\pi}{5}, \sin \frac{\pi}{5}\right) \approx (0.809, 0.588)$

b) $P\left(\frac{29\pi}{4}\right) = P\left(\frac{28\pi}{4} + \frac{\pi}{4}\right) = P(7\pi + \frac{\pi}{4}) = P\left(\frac{5\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

c) $P(122^\circ) = (\cos 122^\circ, \sin 122^\circ) \approx (-0.53, 0.848)$

d) $P\left(-\frac{83\pi}{3}\right) = P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

e) $P\left(\frac{107\pi}{6}\right) = P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

f) $P\left(\frac{119\pi}{7}\right) = P(17\pi) = P(\pi) = (-1, 0)$

11. $r = 2.1 \text{ m}$

$$L = 1.5 \text{ m/s} \times \frac{60 \text{ seconds}}{\text{minute}} \times 2.5 \text{ minutes}$$
$$= 225 \text{ m}$$

$$L = \theta r$$

$$\theta = L/r$$

$$\theta = \frac{225}{2.1}$$

a) $\theta = 107.14 \text{ Radians}$

b) $\frac{n^\circ}{180} = \frac{107.14}{\pi}$

$$n^\circ = \frac{107.14 (180)}{\pi}$$

$$n = 6138.83^\circ$$

12. a) $\sin^4 \theta - \cos^4 \theta = 2\sin^2 \theta - 1$

$$(\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) = 2\sin^2 \theta - 1$$

$$1(\sin^2 \theta - (1 - \sin^2 \theta)) = 2\sin^2 \theta - 1$$

$$\sin^2 \theta - 1 + \sin^2 \theta = 2\sin^2 \theta - 1$$

$$2\sin^2 \theta - 1 = 2\sin^2 \theta - 1$$

f) $(1 + \tan^2 \theta)(1 - \cos^2 \theta) = \sec^2 \theta - 1$

$$(\sec^2 \theta)(\sin^2 \theta) = \sec^2 \theta - 1$$

$$\frac{1}{\cos^2 \theta} \cdot \sin^2 \theta = \sec^2 \theta - 1$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$\sec^2 \theta - 1 = \sec^2 \theta - 1$$

$$b) \frac{\cot^2 x}{1 + \frac{1}{\tan^2 x}} = \cos^2 x$$

$$\frac{\cot^2 x}{1 + \cot^2 x} = \cos^2 x$$

$$\frac{\cot^2 x}{\csc^2 x} = \cos^2 x$$

$$\frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} = \cos^2 x$$

$$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1} = \cos^2 x$$

$$\cos^2 x = \cos^2 x$$

$$c) \frac{\sec^2 \varphi - 1}{1 + \cot^2 \varphi} \cdot \frac{\csc^2 \varphi}{\tan \varphi} \cdot \csc \varphi = \sec \varphi$$

$$\frac{\tan^2 \varphi}{\csc^2 \varphi} \cdot \frac{\csc^2 \varphi}{\tan \varphi} \cdot \csc \varphi = \sec \varphi$$

$$\tan \varphi \cdot \csc \varphi = \sec \varphi$$

$$\frac{\sin \varphi}{\cos \varphi} \cdot \frac{1}{\sin \varphi} = \sec \varphi$$

$$\frac{1}{\cos \varphi} = \sec \varphi$$

$$\sec \varphi = \sec \varphi$$

$$d) \frac{2\cos^2 x - \cos x - 1}{\cos x - 1} = 2\cos x - 1$$

$$\frac{(2\cos x + 1)(\cancel{\cos x - 1})}{\cos x - 1} = 2\cos x - 1$$

$$2\cos x + 1 = 2\cos x - 1$$

$$e) \sin^2 \varphi (1 + \cot^2 \varphi) + \cos^2 \varphi (1 + \tan^2 \varphi) = 2$$

$$\sin^2 \varphi \cdot \csc^2 \varphi + \cos^2 \varphi \cdot \sec^2 \varphi = 2$$

$$1 + 1 = 2$$

$$2 = 2$$

$$g) \frac{\cos^2 \alpha}{1 - \sin \alpha} = 1 + \sin \alpha$$

$$\frac{1 - \sin^2 \alpha}{1 - \sin \alpha} = 1 + \sin \alpha$$

$$\frac{(1 + \sin \alpha)(\cancel{1 - \sin \alpha})}{\cancel{1 - \sin \alpha}} = 1 + \sin \alpha$$

$$1 + \sin \alpha = 1 + \sin \alpha$$

$$h) \frac{\sec^2 \theta \cot \theta}{\csc^2 \theta} = \tan \theta$$

$$\frac{\frac{1}{\cos^2 \theta} \cdot \frac{\cos \theta}{\sin \theta}}{1/\sin^2 \theta} = \tan \theta$$

$$\frac{1}{\cos \theta \sin \theta} \times \frac{\sin^2 \theta}{1} = \tan \theta$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\tan \theta = \tan \theta$$

$$\begin{aligned}
 \text{i) } \sin^4 x - \cos^4 x &= 1 - 2\cos^2 x \\
 (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) &= 1 - 2\cos^2 x \\
 \sin^2 x - \cos^2 x &= 1 - 2\cos^2 x \\
 1 - \cos^2 x - \cos^2 x &= 1 - 2\cos^2 x \\
 1 - 2\cos^2 x &= 1 - 2\cos^2 x
 \end{aligned}$$

$$\begin{aligned}
 \text{j) } \csc^2 \alpha - \cot^2 \alpha &= 1 \\
 \cot^2 \alpha + 1 - \cot^2 \alpha &= 1 \\
 1 &= 1
 \end{aligned}$$

$$13. \text{ a) } \tan x = \sec x$$

$$\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\frac{\sin x - 1}{\cos x} = 0$$

$$\begin{aligned}
 \sin x - 1 &= 0 & \cos x &\neq 0 \\
 \sin x &= 1 & x &\neq \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \\
 x &= \left\{ \frac{\pi}{2} \right\}
 \end{aligned}$$

\therefore No solution

$$\text{b) } \cos \delta + 2 = 3 \cos \delta$$

$$2 = 2 \cos \delta$$

$$1 = \cos \delta$$

$$\delta = \{ 0 \}$$

$$\text{c) } 2 \sin \alpha \cos \alpha = \sqrt{2} \cos \alpha$$

$$2 \sin \alpha \cos \alpha - \sqrt{2} \cos \alpha = 0$$

$$\cos \alpha (2 \sin \alpha - \sqrt{2}) = 0$$

$$\cos \alpha = 0 \quad 2 \sin \alpha - \sqrt{2} = 0$$

$$\alpha = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \quad 2 \sin \alpha = \sqrt{2}$$

$$\sin \alpha = \frac{\sqrt{2}}{2}$$

$$\alpha = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}$$

$$\therefore \alpha = \left\{ \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2} \right\}$$

$$\text{d) } \sin^2 \sigma = 3 \cos^2 \sigma$$

$$0 = 3 \cos^2 \sigma - \sin^2 \sigma$$

$$0 = 3 \cos^2 \sigma - (1 - \cos^2 \sigma)$$

$$0 = 4 \cos^2 \sigma - 1$$

$$1 = 4 \cos^2 \sigma$$

$$\frac{1}{4} = \cos^2 \sigma$$

$$\pm \frac{1}{2} = \cos \sigma$$

$$\sigma = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

13 e) ERROR

$$f) \sec^2 x + 3 \tan x - 11 = 0$$

$$1 + \tan^2 x + 3 \tan x - 11 = 0$$

$$\tan^2 x + 3 \tan x - 10 = 0$$

$$(\tan x - 2)(\tan x + 5) = 0$$

$$\tan x - 2 = 0 \quad \text{or} \quad \tan x + 5 = 0$$

$$\tan x = 2$$

$$\tan x = -5$$

$$x = \tan^{-1}(2)$$

$$x = \tan^{-1}(-5)$$

$$x \doteq 1.11 \text{ rad}$$

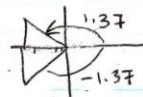
$$x = -1.37$$

$$= 4.91$$



$$x = 2\pi - 1.11$$

$$x \doteq 4.71$$



$$x = 1.37$$

$$x \approx \{ 1.11, 1.37, 4.71, 4.91 \}$$