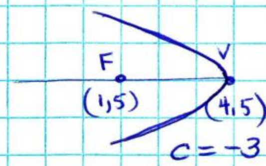
1. $F(6,0)$ $\quad V(4,0)$ $\quad \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$c = 6$
$a = 4$
$a^2 + b^2 = c^2$
$b^2 = 6^2 - 4^2$
$b^2 = 36 - 16$
$b^2 = 20$

Equation : $\dfrac{x^2}{16} - \dfrac{y^2}{20} = 1$

2. $V(4,5)$ $\quad F(1,5)$



$c = -3$

$(y-k)^2 = 4c(x-h)$

$(y-5)^2 = -12(x-4)$

3. $V(0,0)$ $\quad c = 2.5$

$y^2 = 10x$ $\quad$ equation of $\overline{AB}$: $x = 5$

let $x = 5$

$y^2 = 50$

$y = +\sqrt{50}$ $\quad$ or $\quad y = -\sqrt{50}$

$y = 5\sqrt{2}$ $\quad\quad y = -5\sqrt{2}$

$\therefore m\overline{AB} = 2(5\sqrt{2})$

$= 10\sqrt{2}$ units

4. Ellipse : $a = 6$
$b = 8$

$\therefore$ equation $\dfrac{x^2}{36} + \dfrac{y^2}{64} = 1$

Horizontal hyperbola : $a = 6$

slope of asymptote $= \dfrac{24}{18}$

equation of asymptote - $y = \dfrac{24}{18}x$

or $y = \dfrac{8}{6}x$

$y = \dfrac{b}{a}x \Rightarrow a = 6$

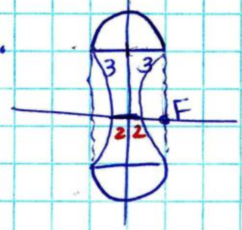
$\therefore b = 8$

Vertical hyperbola :

$y = \dfrac{b}{a}x \Rightarrow y = \dfrac{8}{6}x$

$a = 6$
$b = 8$

5.



$c = 3$

$a = 2$

$a^2 + b^2 = c^2$

$4 + b^2 = 9$

$b^2 = 5$

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

let $x = 3$

$$\frac{9}{4} - \frac{y^2}{5} = 1$$

$$-\frac{y^2}{5} = \frac{4}{4} - \frac{9}{4}$$

$$-\frac{y^2}{5} = -\frac{5}{4}$$

$$y^2 = \frac{25}{4}$$

$$y = \pm \frac{5}{2} \text{ OR } \pm 2.5$$

$$\begin{aligned} m\overline{MP} &= m\overline{AD} + 3 + 3 \\ &= 2(2.5) + 6 \\ &= 5 + 6 \\ &= 11 \text{ cm} \end{aligned}$$

6. $\overline{CD}$: directrix

Mona Lisa (M) : Focus

$\therefore$ Fence : parabola

$\overline{AB}$: x-axis

$\therefore$ $V(0,5)$

$M(0,0)$

$A(-10,0)$

$B(10,0)$

$d(A,M) = d(A,\overline{CD}) = 10\text{m}$

$d(A,B) = 20\text{ m}$

OR $c = -5$

equation $x^2 = -20(y-5)$

let $y = 0$

$$x^2 = -20(-5)$$

$$x^2 = 100$$

$$x = \pm 10$$

$$\therefore d(A,B) = 2(10)$$

$$= 20\text{m}$$

7. Ellipse: $a = \sqrt{146}$ $c^2 = a^2 - b^2$
$b = 5$ $c^2 = 146 - 25$
$c^2 = 121$
$c = 11$

Hyperbola: $b = 5$
$c = 11$
$a^2 + b^2 = c^2$
$a^2 + 25 = 121$
$a^2 = 96$

$$\frac{x^2}{96} - \frac{y^2}{25} = -1$$

let $y = 10$

$$\frac{x^2}{96} - \frac{100}{25} = -1$$
$$\frac{x^2}{96} - 4 = -1$$
$$\frac{x^2}{96} = 3$$
$$x^2 = 288$$
$$x = \pm 12\sqrt{2}$$

Width of sculpture $= 2(12\sqrt{2})$
$= 24\sqrt{2}$ cm

8. $v(0,0)$ Point $(16,8)$

$(x-h)^2 = 4c(y-k)$
$16^2 = 4c(8)$
$256 = 32c$
$8 = c$ directrix: $y = -8$

a) 8 cm

b) let $x = 20$
$x^2 = 32y$
$20^2 = 32y$
$400 = 32y$
$12.5 = y$

Total height: $12.5 + 8$
$= 20.5$ cm

9. a) centre $(0,0)$
$P(0.5, -1.2)$

$$x^2+y^2=r^2$$
$$(0.5)^2+(-1.2)^2=r^2$$
$$0.25+1.44=r^2$$
$$1.69=r^2$$
$$\therefore r=1.3 \text{ km}$$

$$x^2+y^2=1.69$$

b) $V(0,1)$, $P(0.5,2)$

$$(x-h)^2=4c(y-k)$$
$$x^2=4c(y-1)$$
$$(0.5)^2=4c(2-1)$$
$$.25=4c$$
$$0.0625=c$$

$$x^2=0.25(y-1)$$

c) Hotel: $(0, 1.0625)$

$1.0625 < 1.3$
Therefore, the hotel must be evacuated.

d) by substitution

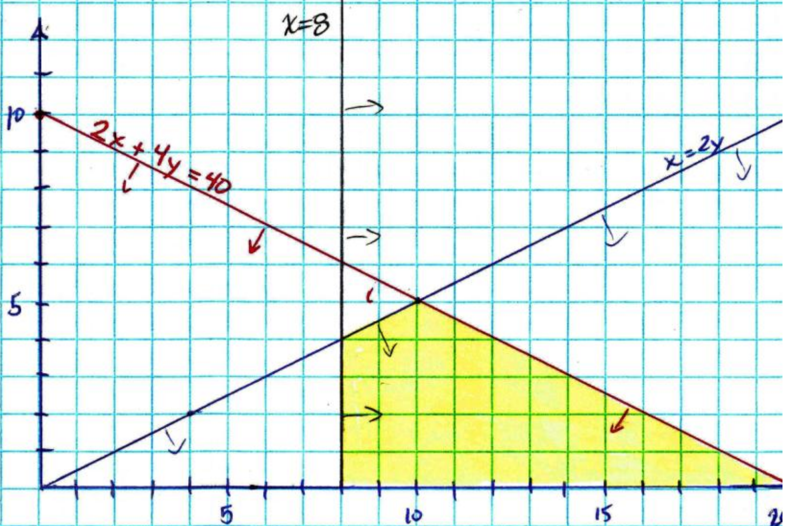
$$0.25(y-1)+y^2=1.69$$
$$0.25y-0.25+y^2=1.69$$
$$y^2+.25y-1.94=0$$
$$y=\frac{-.25\pm\sqrt{0.0625-4(1)(-1.94)}}{2}$$
$$y=\frac{-.25\pm\sqrt{7.8225}}{2}$$
$$y\approx 1.27 \quad \text{OR} \quad y\approx -1.52$$
$$\therefore y=1.27$$

$$x^2=.25(1.27-1)$$
$$x^2=.0684$$
$$x\approx \pm .26$$

$\therefore$ Fences at $(-.26, 1.27)$ and $(.26, 1.27)$

10. x: # of mirrors
y: # of windows

$x \geq 0$
$y \geq 0$
$x \geq 2y$
$2x + 4y \leq 40$
$2x \geq 16$
$\to x \geq 8$



Vertices	$R = 110x + 195y$
(8,0)	\$ 880
(8,4)	1660
(10,5)	2075
(20,0)	2200

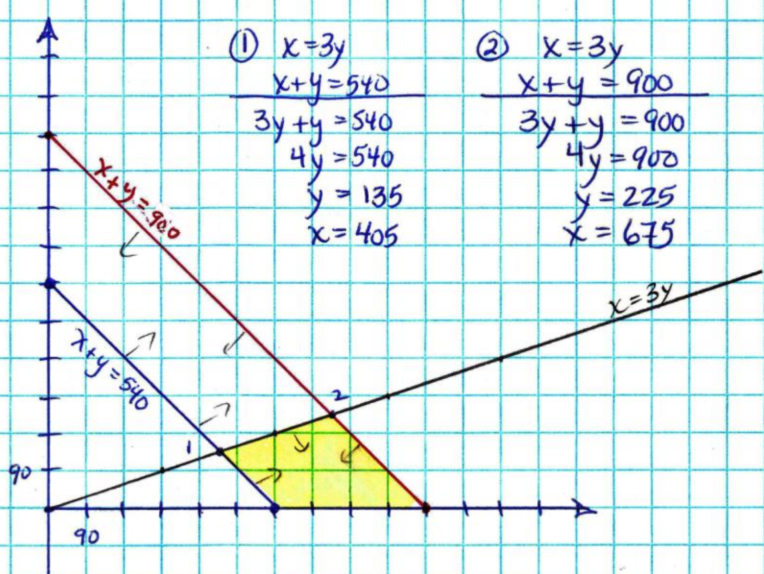
Make 20 mirrors only

11. x: # of à-la-carte dishes
y: # of table d'hôte dishes

$x \geq 0$
$y \geq 0$
$x \geq 3y$
$x + y \geq 540$
$x + y \leq 900$

$.6(12)$ $.75(23)$
$= 7.20$ $= 17.25$

Vertices	$P = 7.2x + 17.25y$
(540,0)	\$ 3 888
(405,135)	5 244.75
(675,225)	8 741.25
(900,0)	6 480



① $x = 3y$
$x + y = 540$
$3y + y = 540$
$4y = 540$
$y = 135$
$x = 405$

② $x = 3y$
$x + y = 900$
$3y + y = 900$
$4y = 900$
$y = 225$
$x = 675$

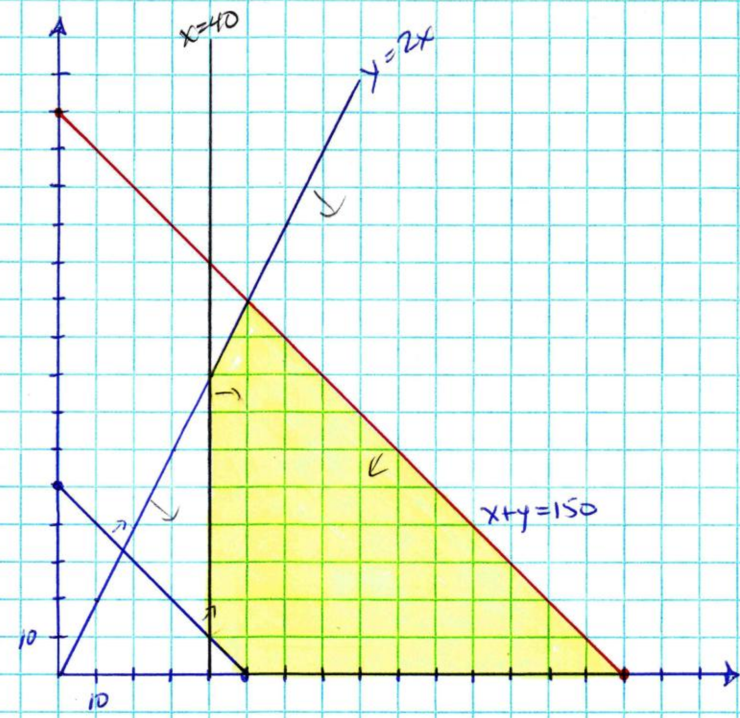
675 à la carte dishes and 225 table d'hôte dishes

12. x: # of graduates
y: # of guests

$x \geq 0$
$y \geq 0$
$x + y \geq 50$
$x + y \leq 150$
$x \geq 40$
$y \leq 2x$

$.25(55) = \$13.75$ } owner
$.65(75) = \$48.75$ }

$\$41.25$ } foundation
26.25 }



Vertices	$R = 13.75x + 48.75y$	$R = 41.25x + 26.25y$
(50, 0)	\$ 687.50	\$ 2062.50
(40, 10)	1037.50	1912.50
(40, 80)	4450	3750
(50, 100)	5562.50	4687.50
(150, 0)	2062.50	6187.50

Owner maximises revenue when 50 graduates and 100 guests attend
Foundation maximises revenue when 150 graduates attend
∴ The owner is wrong

13. a) $V(0,-27)$ $F(0,45)$ $\therefore b=27$, $c=45$

$a^2+b^2=c^2$

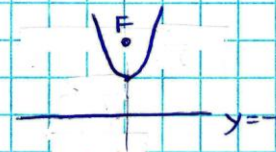
$a^2+27^2=45^2$

$a^2=1296$

$a=36$

$$\frac{x^2}{1296}-\frac{y^2}{729}=-1$$

b) $F(-6,7)$ d: $y=-1$



$7-(-1)=8$

$8\div 2=4$

$\therefore c=4$

$V(-6,3)$

$$(x+6)^2=16(y-3)$$

c) $P(52,96)$ $V(20,0)$ (horizontal)

$a=20$

$\frac{x^2}{400}-\frac{y^2}{b^2}=1$

$\frac{2704}{400}-\frac{9216}{b^2}=1$

$6.76-\frac{9216}{b^2}=1$

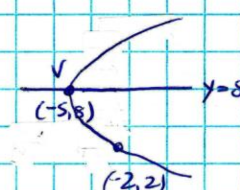
$-\frac{9216}{b^2}=-5.76$

$-9216=-5.76b^2$

$1600=b^2$

$$\frac{x^2}{400}-\frac{y^2}{1600}=1$$

d) $V(-5,8)$ $P(-2,2)$ $y=8$ axis of symmetry



$(y-k)^2=4c(x-h)$

$(2-8)^2=4c(-2+5)$

$36=12c$

$3=c$

$$(y-8)^2=12(x+5)$$

13 e) $c = 13$

$v(5,0)$ horizontal

$a = 5$

$a^2 + b^2 = c^2$

$25 + b^2 = 169$

$b^2 = 144$

$$\frac{x^2}{25} - \frac{y^2}{144} = 1$$

14 a) 1) $v(-40,10)$ $P(-20,-10)$

$(y-k)^2 = 4c(x-h)$

$(-10-10)^2 = 4c(-20+40)$

$400 = 80c$

$5 = c$

$$(y-10)^2 = 20(x+40)$$

$v(-20,10)$ $P(-11,28)$

$(28-10)^2 = 4c(-11+20)$

$324 = 36c$

$9 = c$

$$(y-10)^2 = 36(x+20)$$

2) by substitution

$20(x+40) = 36(x+20)$

$20x + 800 = 36x + 720$

$80 = 16x$

$5 = x$

$(y-10)^2 = 20(45)$

$(y-10)^2 = 900$

$y - 10 = \pm 30$

$y = -20$ or 40

check: $(y-10)^2 = 36(25)$

$(y-10)^2 = 900$ ✓

Answers: $(5,-20)$ and $(5,40)$

b) 1) $v(0,9)$ $b = 9$

$y = \frac{b}{a}x \Rightarrow y = \frac{9}{10}x$

$\therefore a = 10$

$$\frac{x^2}{100} - \frac{y^2}{81} = -1$$

$v(-4,0)$ d: $x = -7$

$c = 3$

$$y^2 = 12(x+4)$$

by substitution $\frac{x^2}{100} - \frac{12(x+4)}{81} = -1$

$81x^2 - 1200x - 4800 = -8100$

$81x^2 - 1200x + 3300 = 0$

$x = \frac{1200 \pm \sqrt{(1200)^2 - 4(81)(3300)}}{162}$

$x = 11.17$ or 3.65

$y^2 = 182.04$

$y = \pm 13.49$

$y^2 = 91.80$

$y = \pm 9.58$

Answers: $(3.65, 9.58)$

$(3.65, -9.58)$

$(11.17, 13.49)$

$(11.17, -13.49)$