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1. Determine the domain and range of each of the following functions.

a) $y = -2x^2 + 4x - 9$

dom = ℝ, ran =]−∞, −7]

b) $y = 4|x - 5| + 8$

dom = ℝ, ran = [8, +∞[

c) $y = \frac{1}{2}\sqrt{-(x - 4)} + 3$

dom =]−∞, 4], ran = [3, +∞[

d) $y = -2\left[\frac{1}{3}(x - 5)\right] + 4$

dom = ℝ,
ran = {y | y = −2m + 4, m ∈ ℤ}

e) $y = \frac{4}{3(x - 1)} + 2$

dom = ℝ \ {1}, ran = ℝ \ {2}

f) $y = -4x + 2$

dom = ℝ, ran = ℝ

2. Determine the zero(s) and the initial value of each of the following functions.

a) $y = -2(x - 4)^2 + 8$

Zeros: 2 and 6, i.v.: −24

b) $y = 3x - 5$

Zero: $\frac{5}{3}$, i.v.: −5

c) $y = \frac{3}{4}\sqrt{x + 1} - 3$

Zero: 15, i.v.: $\frac{-9}{4}$

d) $y = 3\left[\frac{1}{2}(x - 5)\right] + 6$

Zeros: [1, 3[, i.v.: −3

e) $y = \frac{-2}{5(x - 1)} + 4$

Zero: $\frac{11}{10}$, i.v.: $\frac{22}{5}$

f) $y = 3|2x - 1| - 6$

Zeros: $\frac{-1}{2}$ and $\frac{3}{2}$, i.v.: −3

3. Determine over what interval each of the following functions is negative.

a) $y = 2x^2 - 5x - 3$

$\left]-\frac{1}{2}, 3\right[$

b) $y = -7x + 63$

[9, +∞[

c) $y = 2|8 - x| - 12$

[2, 14]

d) $y = \frac{2}{x - 5} + 4$

$\left]\frac{9}{2}, 5\right[$

e) $y = -2\sqrt{6 - x} + 4$

]−∞, 2]

f) $y = -\left[\frac{x}{2}\right] - 3$

[−6, +∞[

4. Determine over what interval each of the following functions is increasing.

a) $y = -3(x - 5)(x + 1)$

]−∞, 2]

b) $y = 2x - 5$

ℝ

c) $y = -[6 - 3x] + 1$

ℝ

d) $y = -3\sqrt{-(x - 1)} + 4$

]−∞, 1]

e) $y = 3|x - 5| + 2$

[5, +∞[

f) $y = \frac{3}{2(x - 1)} + 5$

∅

5. Determine, if it exists, the extremum of each of the following functions.

a) $y = -3x^2 + 12x - 7$

max = 5

b) $y = -2|3 - 2x| + 5$

max = 5

c) $y = -2\sqrt{x} + 7$

max = 7

6. Find the rule of the inverse of each of the following functions.

a) $y = -3x + 8$

$y = -\frac{1}{3}x + \frac{8}{3}$

b) $y = 3\sqrt{2 - x} + 4$

$y = -\frac{1}{9}(x - 4)^2 + 2,\ x \geqslant 4$

c) $y = \frac{3}{2(x - 1)} + 8$

$y = \frac{3}{2(x - 8)} + 1$

7. Consider the following real functions.

$f(x) = 3x - 8$ $\qquad g(x) = 3\sqrt{2x + 1} - 5$ $\qquad h(x) = -2|x - 4| + 12$

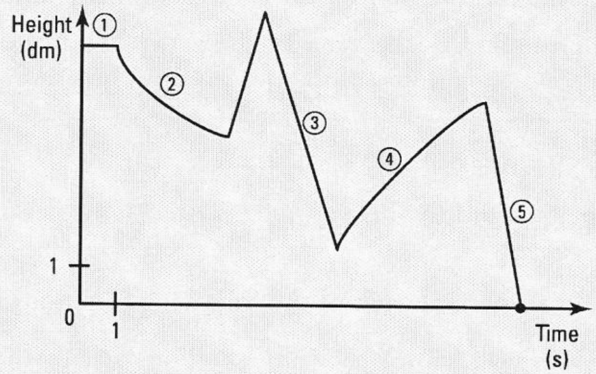
$i(x) = 3(x - 2)^2 + 4$ $\qquad k(x) = \dfrac{2}{x - 5} + 1$ $\qquad l(x) = 3\left[\dfrac{1}{5}(x + 4)\right] - 6$

Determine

a) $f \circ g(4) =$ ***4*** $\qquad$ **b)** $l \circ h(3) =$ ***0*** $\qquad$ **c)** $k \circ i(5) =$ $\boldsymbol{\dfrac{14}{13}}$

d) $f \circ l(0) =$ ***-26*** $\qquad$ **e)** $k \circ f \circ h(2) =$ $\boldsymbol{\dfrac{13}{11}}$ $\qquad$ **f)** $l \circ h(-6) =$ ***-9***

8. The path of a marble in a child's game can be represented by the graph in the Cartesian plane below. Initially, the marble is at a height of 7 dm from the ground.



$$f(x) = \begin{cases} 7 & \text{if } 0 \leq x < 1 \\ \dfrac{3}{x} + 4 & \text{if } 1 \leq x \leq 4 \\ \boldsymbol{a}|x - 5| + 8 & \text{if } 4 \leq x \leq 7 \\ 2\sqrt{x - 7} + \boldsymbol{k} & \text{if } 7 \leq x \leq 11 \\ -5.5x + \boldsymbol{b} & \text{if } 11 \leq x \leq \boldsymbol{t} \end{cases}$$

Determine the duration t of the marble's path.

f(4) = 4.75: $\qquad$ ***a |4 − 5| + 8 = 4.75;*** $\qquad$ ***a = −3.25***

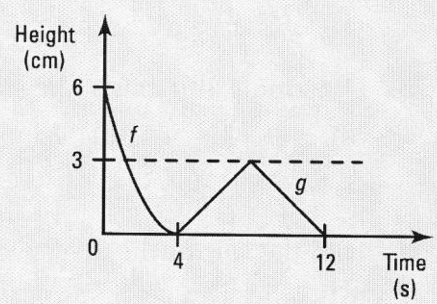
f(7) = 1.5; k = 1.5; $\qquad$ ***f(11) =*** $\boldsymbol{2\sqrt{11 - 7} + 1.5 = 5.5}$

−5.5(11) + b = 5.5; b = 66; −5.5 t + 66 = 0 ⇒ t = 12 s.

9. Aaron is playing an electronic game. The height of a flashing dot on the screen can be modeled by a square root function f from 0 to 4 seconds and by an absolute value function g from 4 to 12 seconds as indicated by the graph on the right.

The starting point of the flashing dot is the vertex of the function f.

Determine at what times the flashing dot is at a height of 1.5 cm.



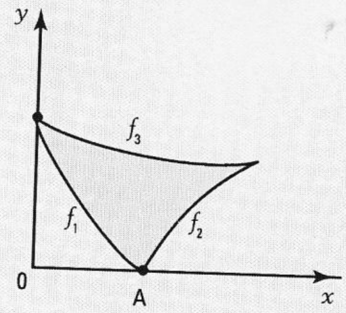
$\boldsymbol{f(x) = -3\sqrt{x} + 6;\ g(x) = -\dfrac{3}{4}|x - 8| + 3}$

The flashing dot is at a height of 1.5 cm at the times t = 2.25 s, t = 6 s and t = 10 s.

10. A company's logo was drawn using the graphs of three square root functions as indicated in the figure on the right.

The rules of the functions f_1 and f_3 are respectively $f_1(x) = -\frac{4}{3}\sqrt{x} + 4$ and $f_3(x) = -\frac{1}{4}\sqrt{x} + 4$.

The x-coordinate of the intersection point of the functions f_2 and f_3 is 16. Knowing that point A is the vertex of the function f_2, what is the rule of the function f_2?



$f_3(16) = 3$; $A(9, 0)$; f_2: $y = a\sqrt{x-9}$

The rule of the function f_2 is: $y = \frac{3}{7}\sqrt{7(x-9)}$

11. The value of one Kandev share fluctuated, over a one-month period, according to the rule of an absolute value function. At the opening of the market, this share was worth \$3.50. Twelve days later, it reaches its maximum value of \$8.

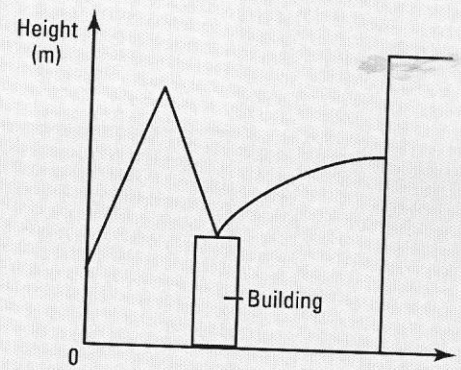
How many days go by between the moment the value of the share is worth \$5 for the first time and the moment it is worth \$2 on its descent?

$y = -\frac{3}{8}|x - 12| + 8$; $-\frac{3}{8}|x - 12| + 8 = 5$; $-\frac{3}{8}|x - 12| + 8 = 2$.

24 days.

12. The graph on the right illustrates a projectile's trajectory thrown from a height of 7 m. After 15 seconds, it reaches its maximum height of 40 m before descending onto the roof of an 18 m high building. The projectile bounces and, 4 seconds later, is at a height of 20 m. The first trajectory follows the model of an absolute value function and the second one follows the model of a square root function whose vertex corresponds to the point where it hits the roof of the building.

The projectile hits the wall of another building at a height of 25 m. How many seconds after the projectile was thrown does it hit the wall of the second building?



$y = -2{,}2\,|x - 15| + 40$; $-2{,}2\,|x - 15| + 40 = 18$; $y = \sqrt{x - 25} + 18$.

The projectile hits the wall of the second building 74 s after it is thrown.