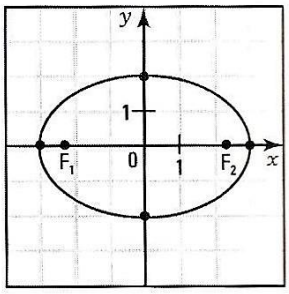
1. Consider the ellipse with equation: $\frac{x^2}{9}+\frac{y^2}{4}=1$.



a) Draw the ellipse in the Cartesian plane on the right.

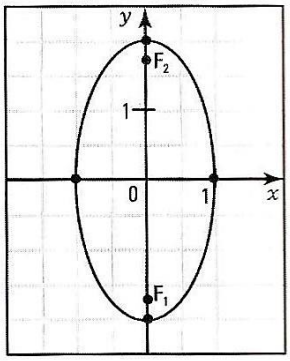
b) Is the major axis horizontal or vertical? ***Horizontal***

c) Find the coordinates of the vertices. ***(–3, 0), (3, 0), (0, –2), (0, 2)***

d) Find the coordinates of the foci F_1 and F_2 and locate the foci in the Cartesian plane. ***$F_1(-\sqrt{5}, 0)$, $F_2(\sqrt{5}, 0)$***

e) If M is a point on the ellipse, determine $d(M, F_1) + d(M, F_2)$. ***6***

2. Consider the ellipse with equation: $\frac{x^2}{1}+\frac{y^2}{4}=1$.



a) Draw the ellipse in the Cartesian plane on the right.

b) Is the major axis horizontal or vertical? ***Vertical***

c) Find the coordinates of the vertices. ***(–1, 0), (1, 0), (0, –2), (0, 2)***

d) Find the coordinates of the foci F_1 and F_2 and locate the foci in the Cartesian plane.
$F_1(-\sqrt{3}, 0)$, $F_2(0, \sqrt{3})$

e) If M is a point on the ellipse, determine $d(M, F_1) + d(M, F_2)$. ***4***

3. a) Explain the steps allowing us to write the standard form of the equation of an ellipse centred at the origin in the general form $ax^2 + by^2 + c = 0$.

$\frac{x^2}{9}+\frac{y^2}{4}=1$ (standard form)

1. $36\left(\frac{x^2}{9}+\frac{y^2}{4}\right)=36$ ***We multiply both sides by the common denominator.***
2. $4x^2 + 9y^2 = 36$ ***We apply distributivity.***
3. $4x^2 + 9y^2 - 36 = 0$ ***We subtract 36 from both sides.***

b) Write the equations of the following ellipses in the general form.

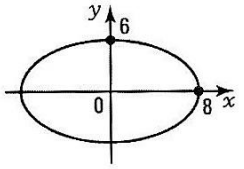
1. $\frac{x^2}{4}+\frac{y^2}{5}=1$ ***$5x^2 + 4y^2 - 20 = 0$***
2. $x^2+\frac{y^2}{9}=1$ ***$9x^2 + y^2 - 9 = 0$***
3. $\frac{x^2}{\frac{4}{9}}+\frac{y^2}{16}=1$ ***$36x^2 + y^2 - 16 = 0$***

4. Complete the following table.

Equation of the ellipse	Position of the major axis	Coordinates of the vertices	Coordinates of the foci	Length of the major axis	Length of the minor axis
$\frac{x^2}{169}+\frac{y^2}{25}=1$	*Horizontal*	*(–13, 0), (13, 0), (0, –5), (0, 5)*	*(–12, 0), (12, 0)*	*26*	*10*
$\frac{x^2}{36}+\frac{y^2}{100}=1$	*Vertical*	*(–6, 0), (6, 0), (0, –10), (0, 10)*	*(0, –8), (0, 8)*	*20*	*12*
$x^2 + 4y^2 - 4 = 0$	*Horizontal*	*(–2, 0), (2, 0), (0, –1), (0, 1)*	*$(\sqrt{-3}, 0)$, $(\sqrt{3}, 0)$*	*4*	*2*
$25x^2 + 4y^2 - 100 = 0$	*Vertical*	*(–2, 0), (2, 0), (0, –5), (0, 5)*	*$(0, -\sqrt{21})$, $(0, \sqrt{21})$*	*10*	*4*

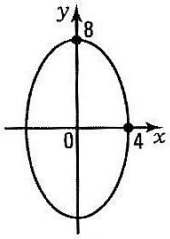
5. Determine the equation of each of the following ellipses.

a)



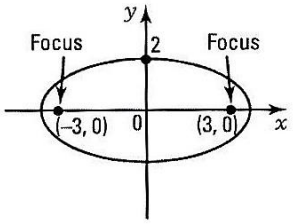
$\frac{x^2}{64} + \frac{y^2}{36} = 1$

b)



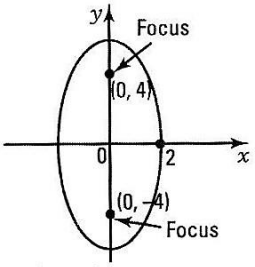
$\frac{x^2}{16} + \frac{y^2}{64} = 1$

c)



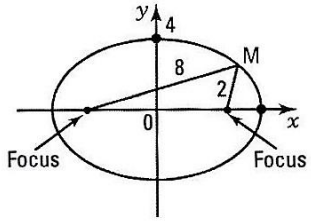
$\frac{x^2}{13} + \frac{y^2}{4} = 1$

d)



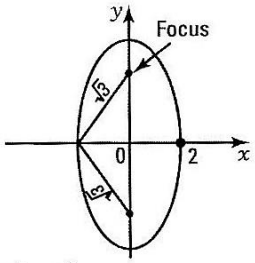
$\frac{x^2}{4} + \frac{y^2}{20} = 1$

e)



$\frac{x^2}{25} + \frac{y^2}{16} = 1$

f)



$\frac{x^2}{4} + \frac{y^2}{3} = 1$

6. Determine the coordinates of the foci of an ellipse centred at the origin whose vertices are the points

a) (6, 0) and (0, 4) $(-\sqrt{20}, 0)$ and $(0, \sqrt{20})$

b) (6, 0) and (0, 7) $(0, -\sqrt{13})$ and $(0, \sqrt{13})$

c) (–10, 0) and (0, –6) $(-8, 0)$ and $(8, 0)$

d) (–4, 0) and (0, –6) $(0, -\sqrt{20})$ and $(0, \sqrt{20})$

7. In each of the following cases, find the equation of the ellipse in the standard form.

a) The major axis of length 20 units is horizontal and the minor axis measures 12 units.

$\frac{x^2}{100} + \frac{y^2}{36} = 1$

b) The major axis of length 16 units is vertical and the minor axis measures 10 units.

$\frac{x^2}{25} + \frac{y^2}{64} = 1$

c) The major axis of length 10 units is horizontal and the focal distance measures 6 units.

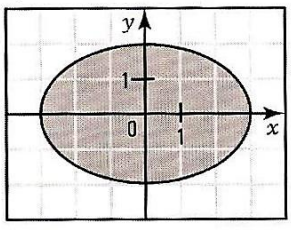
$\frac{x^2}{25} + \frac{y^2}{16} = 1$

d) The major axis of length 20 units is vertical and the focal distance measures 16 units.

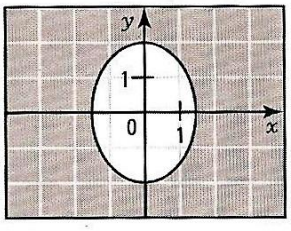
$\frac{x^2}{36} + \frac{y^2}{100} = 1$

8. Represent the following inequalities in the Cartesian plane.

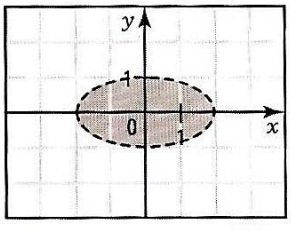
a) $\frac{x^2}{9} + \frac{y^2}{4} \leq 1$



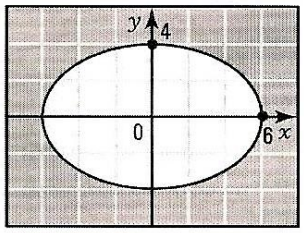
b) $9x^2 + 4y^2 - 16 \geq 0$



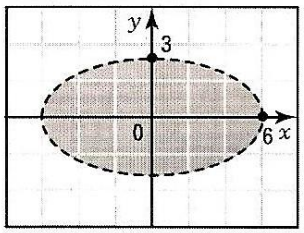
c) $x^2 + 4y^2 - 4 < 0$



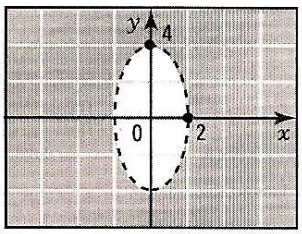
9. Describe each of the following regions using an inequality.

a) 

$\frac{x^2}{36} + \frac{y^2}{16} \geqslant 1$

b) 

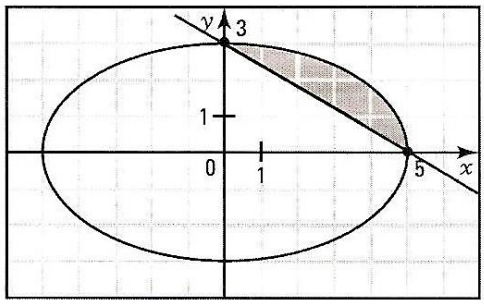
$\frac{x^2}{36} + \frac{y^2}{9} < 1$

c) 

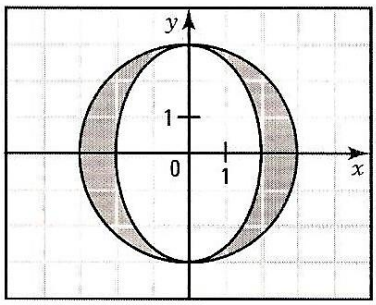
$\frac{x^2}{4} + \frac{y^2}{16} > 1$

10. Represent the following systems in the Cartesian plane.

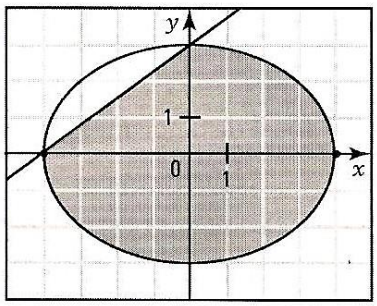
a) $\begin{cases} 9x^2 + 25y^2 - 225 \leqslant 0 \\ 3x + 5y - 15 \geqslant 0 \end{cases}$



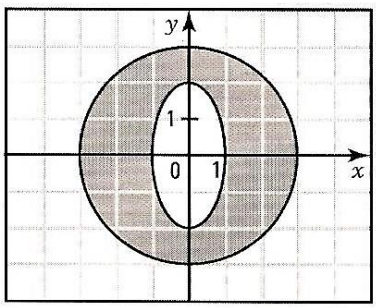
b) $\begin{cases} x^2 + y^2 - 9 \leqslant 0 \\ 9x^2 + 4y^2 - 36 \geqslant 0 \end{cases}$



11. Describe each of the following shaded regions using a system of inequalities.

a) 

$\begin{cases} \frac{x^2}{16} + \frac{y^2}{9} \leqslant 1 \\ 3x - 4y + 12 \geqslant 0 \end{cases}$

b) 

$\begin{cases} x^2 + y^2 \leqslant 9 \\ x^2 + \frac{y^2}{4} \geqslant 1 \end{cases}$

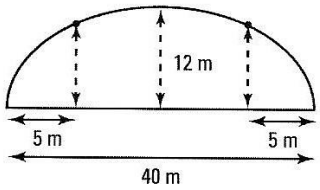
12. Determine the coordinates of the intersection points of the ellipse with equation $x^2 + 4y^2 - 25 = 0$ and the line with equation $x + 2y - 7 = 0$.

Points (3, 2) and $\left(4, \frac{3}{2}\right)$.

13. An art exhibit is presented in a room whose ceiling has the shape of a half-ellipse as shown on the figure on the right.

Spotlights are installed on the ceiling at a horizontal distance of 5 m from the edges of the ceiling.

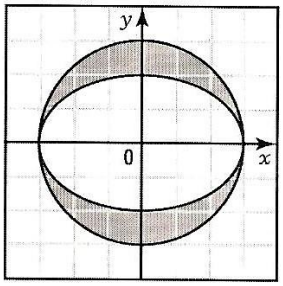
What is the height of the spotlights, as measured from the floor?



Ellipse: $\frac{x^2}{400} + \frac{y^2}{144} = 1$***; when x = 15,*** $y = \pm\sqrt{63}$***. The spotlights are at height 7.94 m.***

14. The circle with equation: $x^2 + y^2 = 9$ and the ellipse with equation: $4x^2 + 9y^2 - 36 = 0$ are represented on the right. Calculate the area of the shaded region.

Note: The area of an ellipse with major axis measuring $2a$ and minor axis measuring $2b$ is equal to πab.



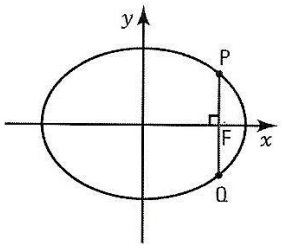
a = 3; b = 2; Area of shaded region = $9\pi - 6\pi = 3\pi$ u^2

15. A race track has an elliptical shape. The length of the major axis is 80 m and the focal distance is 64 m.

What is the length of the minor axis? ***48 m***

16. The measure of the chord perpendicular to the major axis and passing through one of the foci of an ellipse is called **latus rectum**.

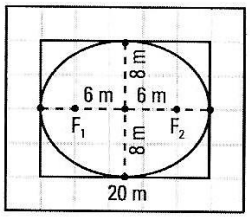
Calculate the measure of the latus rectum of an ellipse defined by the equation $9x^2 + 25y^2 - 225 = 0$.



a = 5; b = 3; c = 4; $y_P = \frac{9}{5}$; $y_Q = -\frac{9}{5}$

Measure of the latus rectum = 3.6 u.

17. A rectangular garden has a perimeter of 72 m and an area of 320 m^2. A landscape architect wants to create a flowerbed shaped as an ellipse in this garden. Using a sketch, give the dimensions of the ellipse and locate the foci so that the flowerbed has maximum area.



Major axis = 20 m; minor axis = 16 m; focal distance = 12 m.