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13. Determine the domain and range of the following rational functions.

a) $y = \frac{3x+2}{x-5}$

dom $= \mathbb{R} \setminus \{5\}$, ***ran f*** $= \mathbb{R} \setminus \{3\}$

b) $y = \frac{-2x+4}{3x-6}$

dom $= \mathbb{R} \setminus \{2\}$, ***ran f*** $= \mathbb{R} \setminus \left\{-\frac{2}{3}\right\}$

c) $y = \frac{5x+4}{2x-3}$

dom $= \mathbb{R} \setminus \left\{\frac{3}{2}\right\}$, ***ran f*** $= \mathbb{R} \setminus \left\{\frac{5}{2}\right\}$

14. Determine the zero (if it exists) and the initial value (if it exists) of the following functions.

a) $y = \frac{3x-2}{x-4}$

Zero: $\frac{2}{3}$, ~~i.v: 3~~ $\frac{1}{2}$

b) $y = \frac{-5x+10}{2x-5}$

Zero: 2, i.v: −2 ✓

c) $y = \frac{-2x-6}{4x}$

✓

Zero: −3, i.v: does not exist

15. Determine over which interval the following functions are positive.

a) $y = \frac{4x+2}{x-3}$

f(x) ≥ 0 over $\left]-\infty, \frac{-1}{2}\right] \cup \left]3, +\infty\right[$

b) $y = \frac{-2x+8}{4x-2}$

f(x) ≥ 0 over $\left]\frac{1}{2}, 4\right]$

16. Study the variation of the following functions.

a) $y = \frac{-4x+9}{x-3}$

f ↗ ***over*** $\mathbb{R} \setminus \{3\}$

b) $y = \frac{2x+5}{3x-2}$

f ↘ ***over*** $\mathbb{R} \setminus \left\{\frac{2}{3}\right\}$

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17. Write the rule of the following rational functions in general form.

a) $y = \frac{3}{2(x-1)} + 4$ $\quad y = \frac{8x-5}{2x-2}$

b) $y = \frac{-2}{5(x-3)} - 1$ $\quad y = \frac{-5x+13}{5x-15}$

18. Write the rule of the following rational functions in standard form.

a) $y = \frac{3x+2}{x-3}$

$y = \frac{11}{x-3} + 3$

b) $y = \frac{4x+3}{2x-6}$

$y = \frac{15}{2(x-3)} + 2$

c) $y = \frac{-2x+5}{3x+4}$

$y = \frac{23}{9\left(x+\frac{4}{3}\right)} - \frac{2}{3}$

19. Consider the rational functions $f(x)=\frac{2x+3}{x-4}$ and $g(x)=\frac{3x+5}{x+3}$.

a) Determine the rule of the composite

1. $g \circ f(x) =$ ***$y = \frac{11x - 11}{5x - 9}$*** 2. $f \circ g(x) =$ ***$y = \frac{9x + 19}{-x - 7}$***

b) What can you say about the composition of a rational function with a rational function?

The composition of a rational function with a rational function is also a rational function.

20. Consider the rational function $y=\frac{5x+4}{x-3}$ (general form).

Justify the steps which enable you to determine the rule of the inverse f^{-1}.

1. Isolate x in the equation $y=\frac{5x+4}{x-3}$.

$y(x - 3) = 5x + 4$ ***Cross products are equal.***

$xy - 3y = 5x + 4$ ***Distributive property of multiplication over subtraction.***

$xy - 5x = 3y + 4$ ***Subtract 5x and add 3y to each side.***

$x(y - 5) = 3y + 4$ ***Factor out x on the left side.***

$x = \frac{3y+4}{y-5}$ ***Isolate the variable x.***

2. Switch the letters x and y to obtain the rule of the inverse.

We get: $y=\frac{3x+4}{x-5}$.

21. Consider the rational function $f(x)=\frac{3x-2}{2x+5}$.

a) Determine the rule of the inverse f^{-1}. ***$f^{-1}(x) = \frac{-5x - 2}{2x - 3}$***

b) Verify that

1. $f \circ f^{-1}(x) = x$ 2. $f^{-1} \circ f(x) = x$

22. Consider the rational function $f(x)=\frac{-2x+3}{4x+1}$.

a) Determine the domain and range of f. ***$\text{dom } f = \mathbb{R} \setminus \left\{-\frac{1}{4}\right\}$, $\text{ran } f = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$***

b) Determine the rule of the inverse f^{-1}. ***$f^{-1}(x) = \frac{-x + 3}{4x + 2}$***

c) Determine the domain and range of the inverse f^{-1} and verify that dom f^{-1} = ran f and ran f^{-1} = dom f.

$\text{dom } f^{-1} = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$, $\text{ran } f^{-1} = \mathbb{R} \setminus \left\{-\frac{1}{4}\right\}$