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$$13 \quad a) \cot^2 x \sin^2 x + \sin^2 x = 1$$

$$\sin^2 x (\cot^2 x + 1) = 1$$

$$\sin^2 x (\csc^2 x) = 1$$

$$\sin^2 x \left( \frac{1}{\sin^2 x} \right) = 1$$

$$1 = 1$$

$$b) \frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} = 1$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 = 1$$

$$c) \frac{1 + \tan^2 x}{\cot^2 x + 1} = \tan^2 x$$

$$\frac{\sec^2 x}{\csc^2 x} = \tan^2 x$$

$$\frac{1/\cos^2 x}{1/\sin^2 x} = \tan^2 x$$

$$1/\cos^2 x \cdot \sin^2 x / 1 = \tan^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

$$\tan^2 x = \tan^2 x$$

$$13 \text{ d)} \frac{\cos^2 x \cdot \tan x}{\cot x} - 1 = -\cos^2 x$$

$$\frac{\cos^2 x \cdot \tan x}{1/\tan x} - 1 = -\cos^2 x$$

$$\cos^2 x \cdot \tan x \cdot \frac{\tan x}{1} - 1 = -\cos^2 x$$

$$\cos^2 x \cdot \tan^2 x - 1 = -\cos^2 x$$

$$\cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x} - 1 = -\cos^2 x$$

$$\sin^2 x - 1 = -\cos^2 x$$

$$-1(1 - \sin^2 x) = -\cos^2 x$$

$$-\cos^2 x = -\cos^2 x$$

$$\text{e)} \frac{\sec x}{\cos x} - 1 = \tan^2 x$$

$$\frac{\sec x}{\cos x} \cdot \frac{1}{\cos x} - 1 = \tan^2 x$$

$$\sec x \cdot \sec x - 1 = \tan^2 x$$

$$\sec^2 x - 1 = \tan^2 x$$

$$\tan^2 x = \tan^2 x$$

$$\text{f)} (1 + \cot^2 x)(1 - \cos^2 x) = 1$$

$$(\csc^2 x)(\sin^2 x) = 1$$

$$\frac{1}{\sin^2 x} \cdot \sin^2 x = 1$$

$$1 = 1$$

$$13 \text{ g) } 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

$$2(1 - \sin^2 a) - 1 = 1 - 2\sin^2 a$$

$$2 - 2\sin^2 a - 1 = 1 - 2\sin^2 a$$

$$1 - 2\sin^2 a = 1 - 2\sin^2 a$$

or,

$$2\cos^2 a - 1 = 1 - 2\sin^2 a$$

$$2\cos^2 a - 1 = 1 - 2(1 - \cos^2 a)$$

$$2\cos^2 a - 1 = 1 - 2 + 2\cos^2 a$$

$$2\cos^2 a - 1 = 2\cos^2 a - 1$$

$$\text{h) } \tan x + \cot x = \sec x \cdot \csc x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \sec x \cdot \csc x$$

$$\left(\frac{\sin x}{\sin x}\right) \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \left(\frac{\cos x}{\cos x}\right) = \sec x \cdot \csc x$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x} = \sec x \cdot \csc x$$

$$\frac{1}{\sin x \cdot \cos x} = \sec x \cdot \csc x$$

$$\frac{1}{\sin x} \cdot \frac{1}{\cos x} = \sec x \cdot \csc x$$

$$\csc x \cdot \sec x = \sec x \cdot \csc x$$

$$14. \text{ a) } \sec x - \cos x = \sin x \tan x$$

$$\frac{1}{\cos x} - \cos x = \sin x \tan x$$

$\cos x$

$$\frac{1}{\cos x} - \cos x \left( \frac{\cos x}{\cos x} \right) = \sin x \tan x$$

$$\frac{1 - \cos^2 x}{\cos x} = \sin x \tan x$$

$$\frac{\sin^2 x}{\cos x} = \sin x \tan x$$

$$\frac{\sin x \cdot \cancel{\sin x}}{\cos x} = \sin x \tan x$$

$$\sin x \cdot \tan x = \sin x \cdot \tan x$$

$$\text{b) } \frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$$

$$\left( \frac{1 - \sin x}{1 - \sin x} \right) \frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$$

$$\frac{1 - \sin^2 x}{\cos x (1 - \sin x)} = \frac{\cos x}{1 - \sin x}$$

$$\frac{\cos^2 x}{\cos x (1 - \sin x)} = \frac{\cos x}{1 - \sin x}$$

$$\frac{\cos x}{1 - \sin x} = \frac{\cos x}{1 - \sin x}$$

$$c) \frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$$

diff.  
of squares

$$\frac{1 - \cos^2 x}{1 - \cos x} = 1 + \cos x$$

$$\frac{(1 + \cos x)(1 - \cos x)}{1 - \cos x} = 1 + \cos x$$

$$1 + \cos x = 1 + \cos x$$

$$d) (1 + \tan x)^2 + (1 - \tan x)^2 = 2 \sec^2 x$$

$$1 + 2\tan x + \tan^2 x + 1 - 2\tan x + \tan^2 x = 2 \sec^2 x$$

$$2 + 2\tan^2 x = 2 \sec^2 x$$

$$2(1 + \tan^2 x) = 2 \sec^2 x$$

$$2 \sec^2 x = 2 \sec^2 x$$

$$e) (1 + \tan x)(1 - \tan x) - (1 + \cot x)(1 - \cot x) = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$(1 - \tan^2 x) - (1 - \cot^2 x) = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$-\tan^2 x + \cot^2 x = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$\cot^2 x - \tan^2 x = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$\frac{1}{\tan^2 x} - \tan^2 x = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$\frac{1}{\tan^2 x} - \tan^2 x \left( \frac{\tan^2 x}{\tan^2 x} \right) = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$\frac{1}{\tan^2 x} - \frac{\tan^4 x}{\tan^2 x} = \frac{1 - \tan^4 x}{\tan^2 x}$$

$$\frac{1 - \tan^4 x}{\tan^2 x} = \frac{1 - \tan^4 x}{\tan^2 x}$$