1. A driving school training program includes a minimum of four theory courses. A student must take at least twice as many practical courses as theory courses. The maximum number of courses allowed is 20.

Let x represent the number of theory courses and y the number of practical courses.

The constraints translating the situation above are given below.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 20$$
$$x \geq 4$$
$$\underline{\quad ? \quad}$$

Which is the missing constraint?

A) $y \geq 2x$ C) $2y \geq x$

B) $y \leq 2x$ D) $2y \leq x$

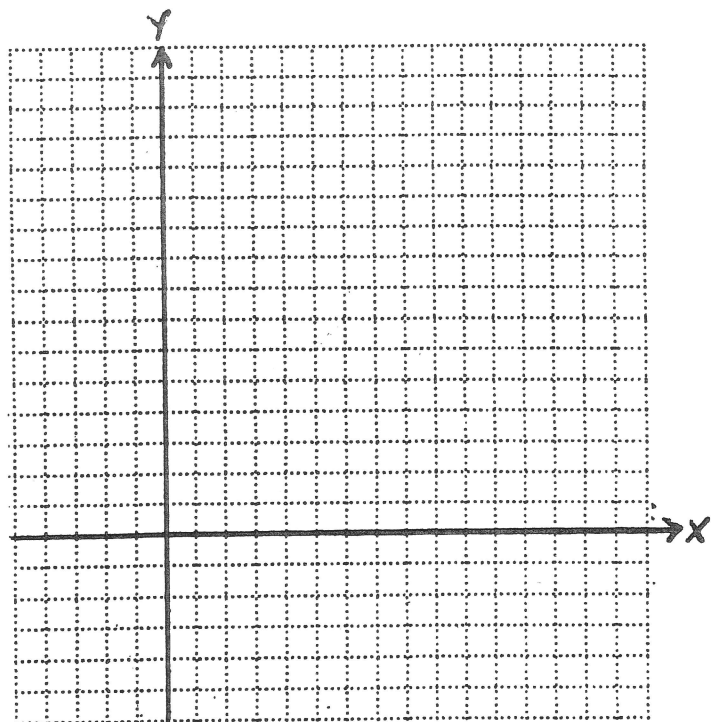2. Draw the polygon of constraints associated with the following system of inequalities:

$$x \geq 0$$
$$y \geq 0$$
$$y \leq -2x + 16$$
$$y \leq \frac{x}{2} + 8$$
$$y \geq 2$$

3. Darcy has red fish (x) and moon fish (y) in his aquarium.

To maintain a healthy environment for the two types of fish, Darcy must respect the constraints below:

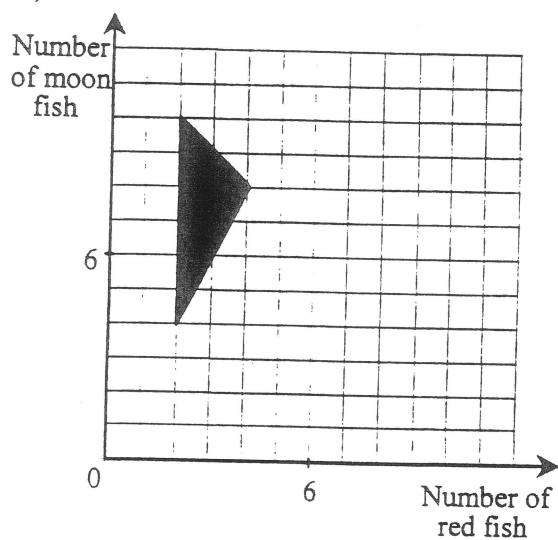$$x \geq 0 \qquad x \geq 2 \qquad x + y \leq 12$$

$$y \geq 0 \qquad y \leq 8 \qquad x \leq 2y$$
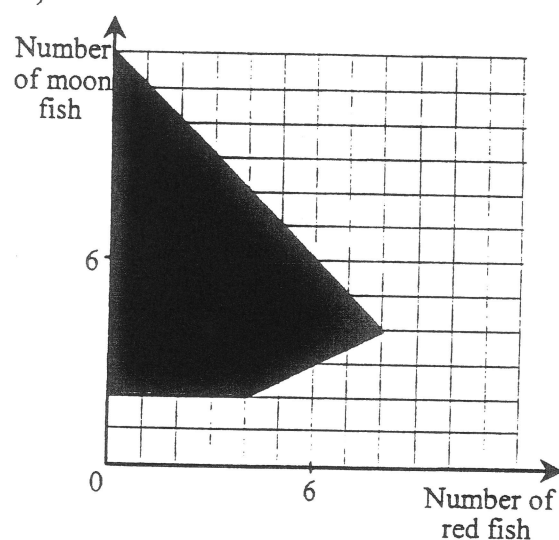
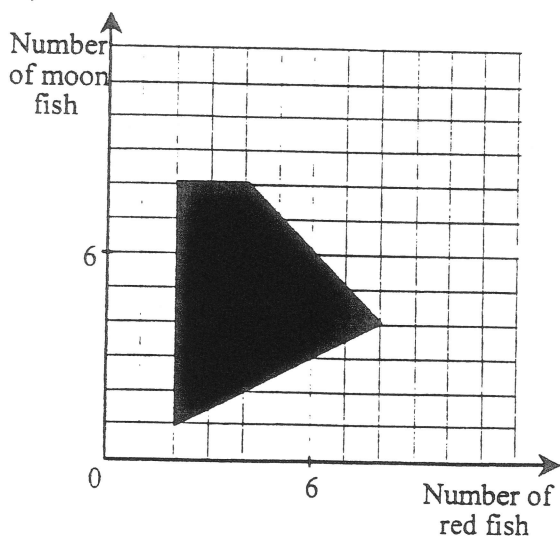**Which of the following polygons of constraints represents the situation above?**

A)

B)
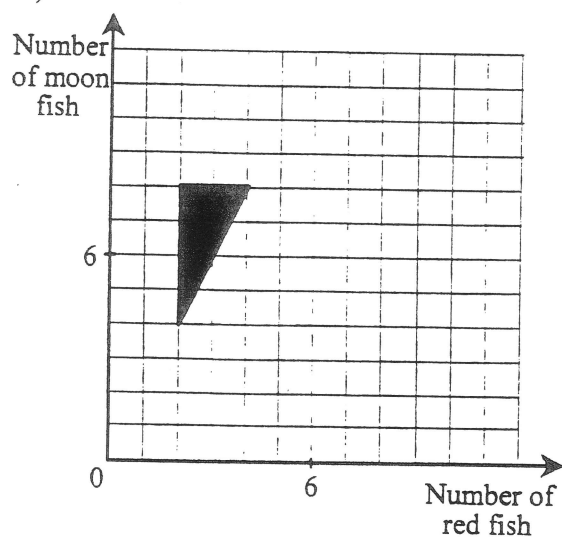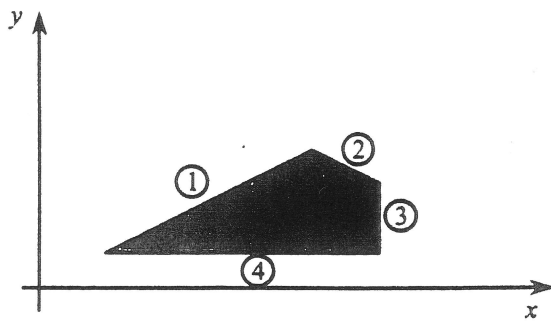

C)

D)

4. Given a polygon of constraints and the corresponding system of inequalities.

Each side of the polygon and its corresponding relation are identified by the same number.

① $x \geq 2y$

② $x + 2y \leq 160$

③ $x \leq 100$

④ $y \geq 10$

Which of the following ordered pairs is a vertex of the polygon of constraints?

A)   (10, 20)

B)   (40, 80)

C)   (100, 30)

D)   (140, 10)

5. **Which of the following statements are FALSE?**

1. In a polygon of constraints, the coordinates of the point farthest from the origin maximize the value of the function to be optimized.

2. In a polygon of constraints, the coordinates of the lowest point minimize the value of the function to be optimized.

3. Any optimization problem has one and only one optimal solution.

A)   1 and 2 only

B)   1 and 3 only

C)   2 and 3 only

D)   1, 2 and 3

6. A pastry chef has been hired to bake several vanilla and chocolate cakes. He needs at least 4 vanilla cakes and 3 chocolate cakes. However, because of lack of space, he cannot make more than 11 cakes. The profit on a vanilla cake is $5 while on a chocolate cake it is $6.

How many cakes of each type must the pastry chef make in order to maximize his profits?

A)   4 vanilla cakes and 7 chocolate cakes

B)   4 vanilla cakes and 3 chocolate cakes

C)   8 vanilla cakes and 7 chocolate cakes

D)   8 vanilla cakes and 3 chocolate cakes

7. Claudia is the owner of a small maple syrup company. Each spring, she sells the syrup to her friends. The syrup is sold in two formats: 1 litre and 3 litres.

The 1-L container sells for $8 and the 3-L container sells for $20.

The polygon below represents the constraints of this situation.
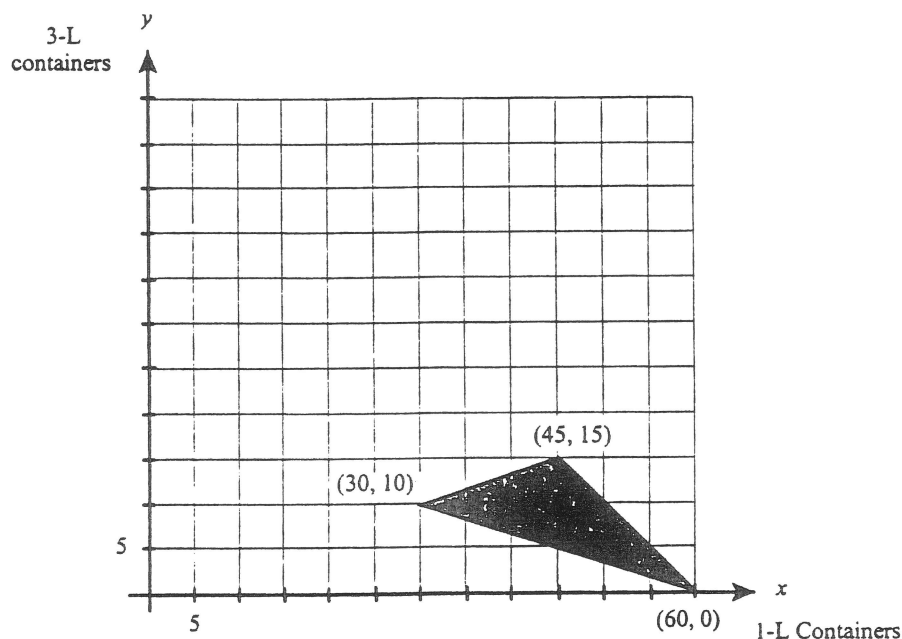
How many containers of each format must Claudia sell to maximize her profits?

A)   30 containers of 1 litre and 10 containers of 3 litres

B)   45 containers of 1 litre and 15 containers of 3 litres

C)   60 containers of 1 litre

D)   60 containers of 3 litres

8. As a fund-raising activity, a school Student Council decides to sell jerseys: T-shirts and fleece vests. According to a poll, the student council can expect to sell the following:
   - a minimum of 150 jerseys;
   - a maximum of 450 jerseys;
   - at least twice as many T-shirts as fleece vests.

The Student Council can make a profit of $5 on each T-shirt sold and $10 on each fleece vest.

How many jerseys of each type must the Student Council sell to make the maximum profit?

9. A speciality store sells two designer-style hats: Colomb and Cartier. The owner cannot keep any more than 60 hats in the store. However, he must always have at least 10 hats of each style. He sells at least twice as many Colomb hats as Cartier hats.

His profit on a Colomb hat is $15 and on a Cartier hat is $25.

How many hats of each style must the owner sell to maximize his profit?

10. A future housing development will consist of duplexes (houses with two apartments) and quadruplexes (houses with four apartments).

The promoters of this development face different constraints. The following polygon represents the different possible combinations of duplexes and quadruplexes that can be built.
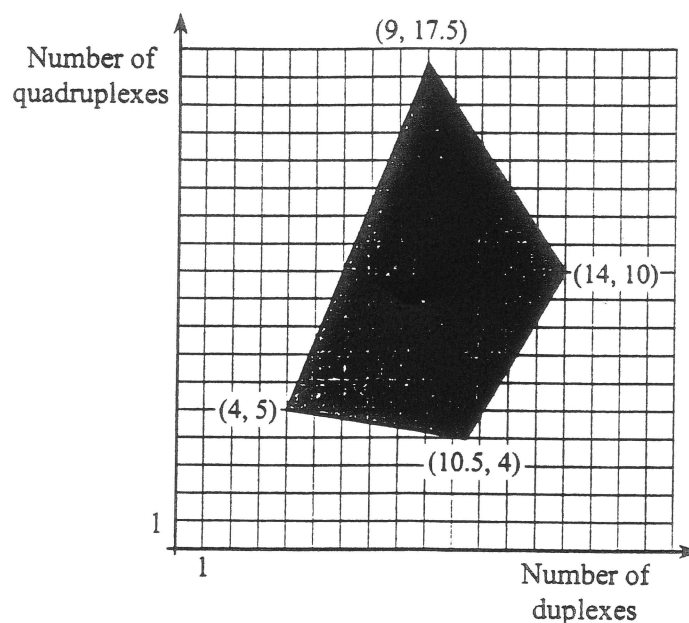
The number of apartments (N) that will be available once the project is completed is determined by the following rule:

$$N = 2x + 4y$$

where x: number of duplexes
                 y: number of quadruplexes

How many duplexes and quadruplexes must the promoters build to maximize the number of apartments?

A)   9 duplexes and 17 quadruplexes   C)   10 duplexes and 16 quadruplexes

B)   9 duplexes and 18 quadruplexes   D)   14 duplexes and 10 quadruplexes

11. A company has launched a new shampoo and a new conditioner. These products are sold in 500-mL bottles.

Below are the results of a market study.

- Every month, the number of bottles of shampoo sold will be greater than or equal to the number of bottles of conditioner sold.
- The company will sell no more than 5000 bottles of these new products every month.
- The company will sell at least 1500 bottles of shampoo a month.

The team that conducted the market study suggested two combinations of selling prices for these products:
- $3.00 per bottle of shampoo and $3.00 per bottle of conditioner;
- $2.80 per bottle of shampoo and $3.10 per bottle of conditioner.

Which combination of prices should the company choose to maximize its revenue?

12. The constraints for a situation are represented by the following system of inequalities:

$$x \geq 0$$
$$y \geq 0$$
$$2y \geq x$$
$$x \geq 2$$
$$10x + 8y \leq 80$$

Which polygon of constraints represents this situation?

A)

C)

B)

D)
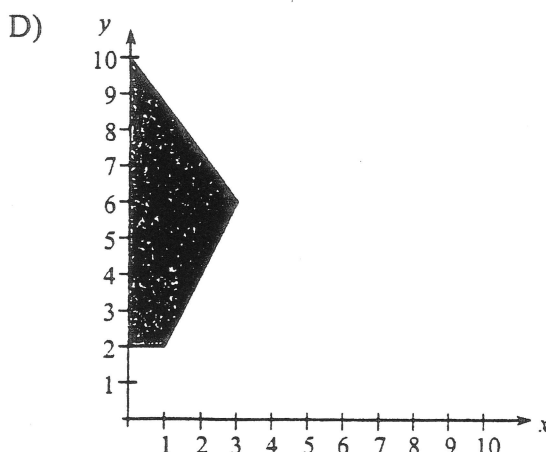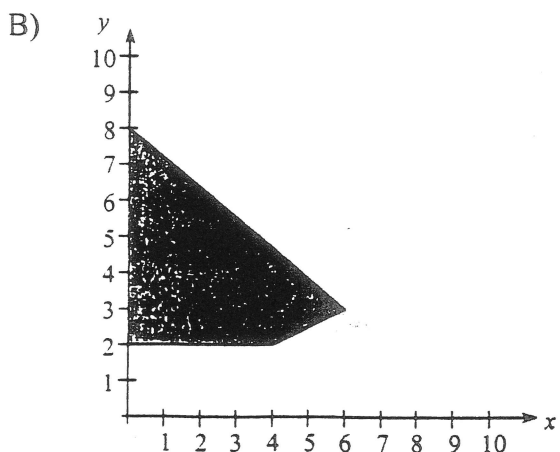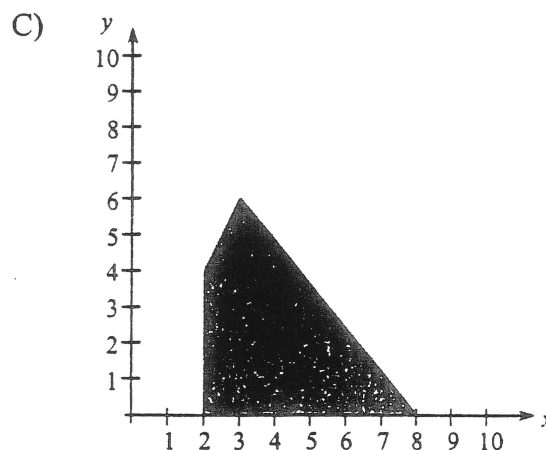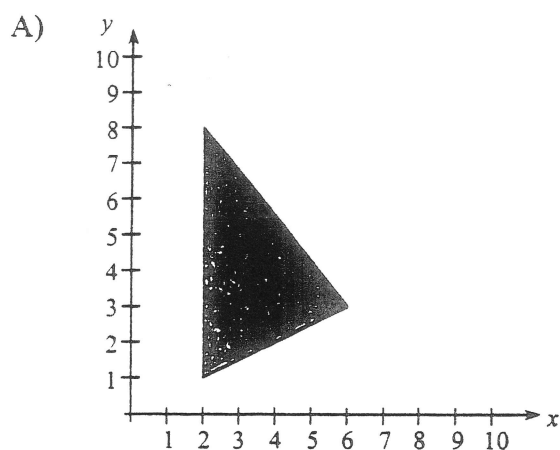
13. The system of inequalities and the polygon of constraints below are associated with an optimization situation. Each side of the polygon and its related inequality are identified by the same number.

① $3x + y \geq 24$
② $x + y \leq 20$
③ $y \geq x - 8$
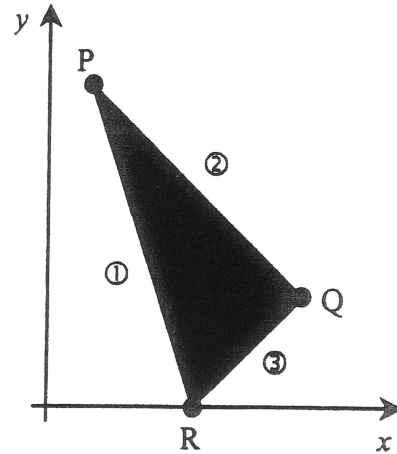
What are the coordinates of vertex P of this polygon of constraints?

14. The system of inequalities and the polygon of constraints below are associated with an optimization situation. Each side of the polygon and its related inequality are identified by the same number.
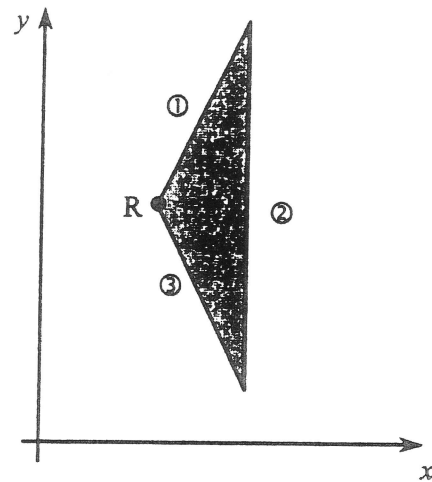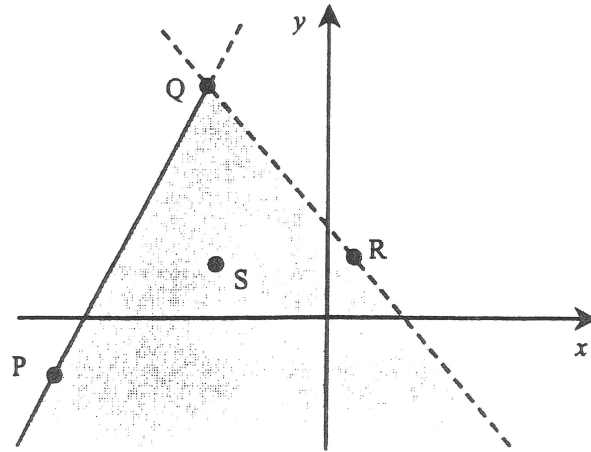
① $y \leq 2x$
② $x \leq 16$
③ $y \geq -2x + 36$

What are the coordinates of vertex R of this polygon of constraints?

15. The solutions for a system of inequalities are represented in the following Cartesian plane. Points P, Q, R and S are also shown on the graph.

Which of these four points represent solutions for this system of inequalities?

A) P and S only   C) P, Q and R only

B) Q and S only   D) P, Q, R and S

16. The constraints for an optimization situation are represented by the following system of inequalities:

$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 6$$
$$3x + 2y \leq 24$$
$$x \leq 7$$
$$x \geq y$$

Which one of the following ordered pairs is a solution for this system of inequalities?

A) (1, 6)   C) (6, 4)

B) (3, 3)   D) (8, 0)

17. Jerry is a university student. He rents an apartment for $360 a month. Every month, he spends some of his free time giving computer lessons. He earns $12 for each lesson given to a child and $20 for each lesson given to an adult.

His availability and the demand for lessons create certain constraints, which are represented by the inequalities and the polygon of constraints given below. Each side of the polygon and its corresponding inequality are identified by the same number.

$$x \geq 0$$
$$y \geq 0$$
① $$y \geq 2x$$
② $$x \geq 5$$
③ $$x + y \leq 27$$
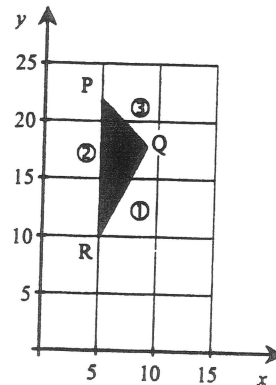

x: the number of lessons given to children per month
y: the number of lessons given to adults per month

Can Jerry always be sure that these computer lessons will pay him enough money every month to cover his rent?

Show all your work and explain your final answer.

18. The student council of a school is organizing a fashion show, which will be held in the school auditorium. The tickets will cost $4 for students at the school and $6 for the general public.

The following polygon of constraints reflects the different constraints faced by the student council.
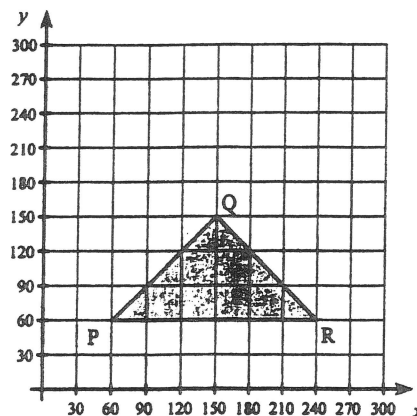
x: number of tickets for students at the school
y: number of tickets for the general public

A runway will be built so that the models can walk down the middle of the auditorium. The runway will reduce the seating capacity. As a result, only a maximum of 240 tickets can be sold.

By how much will the maximum possible revenue decrease once the runway is built?

Show all your work.

19. Jenny has two part-time jobs for the summer. She cleans a senior citizens' centre and also works in a grocery store.

Jenny makes $9 an hour when she works at the senior citizens' centre and $8 an hour when she works at the grocery store.

Every week, there are different constraints on the number of hours she can devote to each job. This situation is represented by the following polygon of constraints:
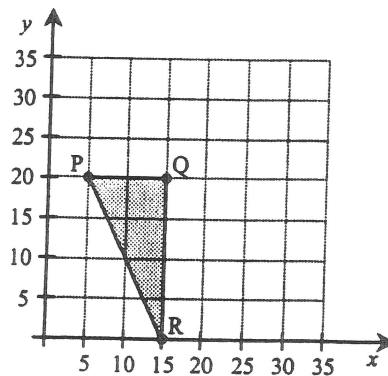
Coordinates of the vertices of the polygon
P (5, 20)
Q (15, 20)
R (15,0)

x : number of hours worked in the senior citizens' centre
y : number of hours worked in the grocery store

Since Jenny will be playing in a soccer tournament this week, she can work at most 30 hours.

By how much will Jenny's maximum possible income decrease because of this additional constraint?

Show all your work.

20. Vincent works for a company that makes storage racks for compact disks. Each week, he ‹ time between assembly work and finishing work.

The polygon of constraints below represents the different constraints that Vincent faces.
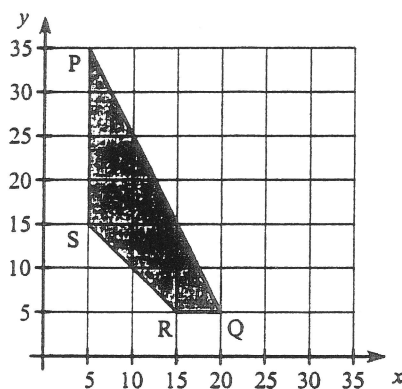
Coordinates of the vertices of the polygon of constraints
P(5, 35) Q(20, 5) R(15, 5) S(5, 15)

x: number of hours spent on assembly work each week
y: number of hours spent on finishing work each week

Vincent is told that from now on he faces the following additional constraint: the number ‹ spent on finishing work must be less than or equal to the number of hours spent on assemb

He makes $10 an hour for assembly work and $8 an hour for finishing work.

By how many dollars does this constraint decrease Vincent's maximum possible weekly ir

Show all your work.