1. $(0,-1)$ minimum: vertex $(8,-5)$

$y = a|x-h|+k$

$-1 = a|0-8|-5$

$4 = a|-8|$

$4 = 8a$

$\frac{1}{2} = a$ $\therefore\ y = \frac{1}{2}|x-8|-5$

$y \leq -3$

$\frac{1}{2}|x-8|-5 \leq -3$

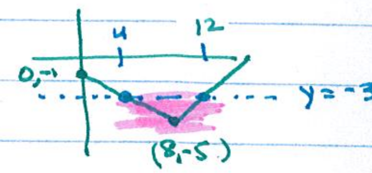
$\frac{1}{2}|x-8|-5 = -3$

$\frac{1}{2}|x-8| = 2$

$|x-8| = 4$

$x-8 = \pm 4$

$x = \{4, 12\}$



Answer: $12-4 = \boxed{8 \text{ hours}}$

2. $(0,25)$ $(5,10)$: vertex (minimum)

$y = a|x-5|+10$

$25 = a|0-5|+10$

$15 = a|-5|$

$15 = 5a$

$3 = a$ $y = 3|x-5|+10$

$x = 12$ $y = 3|12-5|+10$

$y = 3|7|+10$

$y = 3(7)+10$

$y = 21+10$

$y = 31$

Answer: $\boxed{\$31.}$

3 $f(x) = \frac{5}{4}|x-10| + 6$

Top of glass: $\underline{x = 10 \pm 4 = \{6, 14\}}$

$f(14) = \frac{5}{4}|14-10| + 6$

$= \frac{5}{4}|4| + 6$

$= \frac{5}{4}(4) + 6$

$= 5 + 6$

$= \underline{11}$ The glass is 11 cm high

Gold leaf strip: $11 - 2 = 9$ cm

let $y = 9 \Rightarrow 9 = \frac{5}{4}|x-10| + 6$

$3 = \frac{5}{4}|x-10|$

$2.4 = |x-10|$

$\pm 2.4 = x - 10$

$\{7.6, 12.4\} = x$

Answer: The diameter of the gold leaf is $12.4 - 7.6$

$= \boxed{4.8 \text{ cm}}$

4. $(0, 28)$ maximum (vertex) : $(10, 43)$

$y = a|x-h| + k$

$28 = a|0-10| + 43$

$-15 = a|-10|$

$-15 = 10a$

$-1.5 = a$ $\underline{y = -1.5|x-10| + 43}$

let $y = 25$

$25 = -1.5|x-10| + 43$

$-18 = -1.5|x-10|$

$12 = |x-10|$

$\pm 12 = x - 10$

$\{-2, 22\} = x$

reject, time cannot be negative

Answer: $\boxed{\text{Week } 22}$

5. $\sin A \cot A + 2\cos^2 A = 1 \qquad A \in [0, 2\pi]$

$$2\cos^2 A + \cancel{\sin A} \cdot \frac{\cos A}{\cancel{\sin A}} = 1$$

$$2\cos^2 A + \cos A = 1$$

$$2\cos^2 A + \cos A - 1 = 0$$

(a trinomial of the form $2x^2 + x - 1 = 0$)

by factoring

$a \cdot c = -2$, $b = 1$ $\Big\}$ $m = 2,\ n = -1$

$$2x^2 + 2x - x - 1 = 0$$

$$2x(x+1) - 1(x+1) = 0$$

$$(x+1)(2x-1) = 0$$

$x = \cos A$

$\therefore \quad \cos A + 1 = 0$ or $2\cos A - 1 = 0$

$\cos A = -1 \qquad 2\cos A = 1$

$\downarrow \qquad \cos A = 1/2$

$A = \pi \qquad A = \{\pi/3, \frac{5\pi}{3}\}$

quadratic formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{4}$$

$$x = \frac{-1 \pm \sqrt{9}}{4}$$

$$x_1 = \frac{-1-3}{4} = \frac{-4}{4} = -1$$

$$x_2 = \frac{-1+3}{4} = \frac{2}{4} = 1/2$$

$x = \cos A$

$\therefore \cos A = -1$ or $\cos A = 1/2$

Answer: $\boxed{A = \{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}}$

6. $V(t) = -25|t-8| + 65$

$$-25|t-8| + 65 \geq 35$$

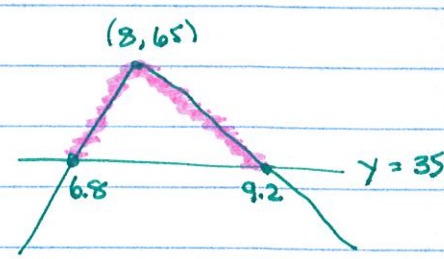
$$-25|t-8| + 65 = 35$$

$$-25|t-8| = -30$$

$$|t-8| = 1.2$$

$$t - 8 = \pm 1.2$$

$$t = \{6.8, 9.2\}$$



Answer: $9.2 - 6.8 = \boxed{2.4 \text{ hours}}$

7. a: slope of $\overline{AB}$ $= \frac{7-4}{0-(-2)} = \frac{3}{2}$ $\therefore$ $a = -\frac{3}{2}$ (opens downward)

equation of $\overline{AB}$: $y = \frac{3}{2}x + b$ B(0,7)

$y = \frac{3}{2}x + 7$

equation of right ray: $y = -\frac{3}{2}x + b$ C(4,5)

$5 = -\frac{3}{2}(4) + b$

$5 = -6 + b$

$11 = b$

$y = -\frac{3}{2}x + 11$

Vertex: by comparison

$\frac{3}{2}x + 7 = -\frac{3}{2}x + 11$

$\frac{6}{2}x + 7 = 11$

$3x = 4$

$x = 4/3$

$y = \frac{3}{2}(4/3) + 7$ // $y = -\frac{3}{2}(\frac{4}{3}) + 11$

$y = 2 + 7$ // $y = -2 + 11$

$y = 9$ // $y = 9$

Rule: $y = -\frac{3}{2}\left|x - \frac{4}{3}\right| + 9$

let $y = 3$ $3 = -\frac{3}{2}|x - 4/3| + 9$

$-6 = -\frac{3}{2}|x - 4/3|$

$4 = |x - 4/3|$

$(4 = \frac{12}{3})$ $\pm 4 = x - 4/3$

$\{-8/3, 16/3\} = x$

$\therefore$ base of triangle $= x_2 - x_1$

$= 16/3 - (-8/3)$

$= \frac{24}{3}$

$= 8$ units

height of triangle $= 9 - 3 = 6$

Answer: Area of front of birdhouse $= \frac{8 \cdot 6}{2} = \boxed{24 \text{ square units}}$

8. parabola $y^2 - x + 8y + 17 = 0$ convert to standard

$(y-k)^2 = 4c(x-h)$

complete the square

$-8 \div 2 = -4$

$(-4)^2 = \underline{\underline{16}}$ add

$$y^2 - 8y = x - 17$$

$$y^2 - 8y + 16 = x - 17 + 16$$

$$(y-4)^2 = x - 1$$

vertex: $(1, 4)$

let $x = 5$

$$y^2 - 5 + 8y + 17 = 0$$

$$y^2 - 8y + 12 = 0$$

$$(y-6)(y-2) = 0$$

$$y - 6 = 0 \quad \text{or} \quad y - 2 = 0$$

$$y = \{2, 6\}$$

Circle: centre $(1, 4)$ Point $(5, 2)$ or $(5, 6)$

$$(x-1)^2 + (y-4)^2 = r^2$$

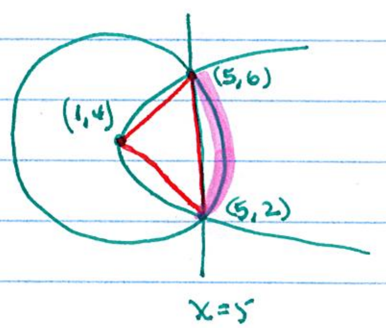
$$(5-1)^2 + (6-4)^2 = r^2$$

$$4^2 + 2^2 = r^2$$

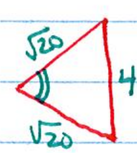
$$16 + 4 = r^2$$

$$20 = r^2$$

$$(x-1)^2 + (y-4)^2 = 20$$



Degree of arc = degree of central angle



$$\theta = \cos^{-1}\left(\frac{4^2 - (\sqrt{20})^2 - (\sqrt{20})^2}{-2(\sqrt{20})(\sqrt{20})}\right)$$

$$\theta = \cos^{-1}\left(\frac{-24}{-40}\right)$$

$$\theta = \cos^{-1}(0.61$$

$$\theta = 53.13^\circ$$

9. Larger circle : $x^2+y^2-12x-16y-44=0$

convert : $x^2-12x \quad + y^2-16y = 44$

$-12 \div 2 = -6 \qquad -16 \div 2 = -8$

$(-6)^2 = 36 \qquad (-8)^2 = 64$

$x^2-12x+36 + y^2-16y+64 = 44+36+64$

$(y-6)^2 + (y-8)^2 = 144$

centre $(6,8)$ $\quad r = 12$

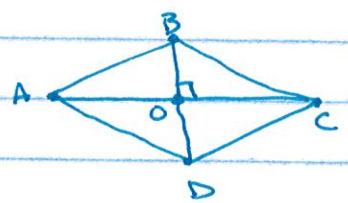
Answer: Smaller circle : centre $(6,8)$ $\quad r = 12-8 = 4$

equation $\boxed{(x-6)^2+(y-8)^2 = 16}$

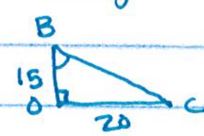
10. $\dfrac{x^2}{400} + \dfrac{y^2}{225} = 1$ $\qquad a = \sqrt{400} = \pm 20$

$b = \sqrt{225} = \pm 15$

$A(-20,0) \quad B(0,15) \quad C(20,0) \quad D(0,-15)$



All four triangles are congruent



$m\angle CBO$: $\tan\theta = \dfrac{20}{15}$

$\theta = \tan^{-1}(4/3)$

$\theta = 53.13°$

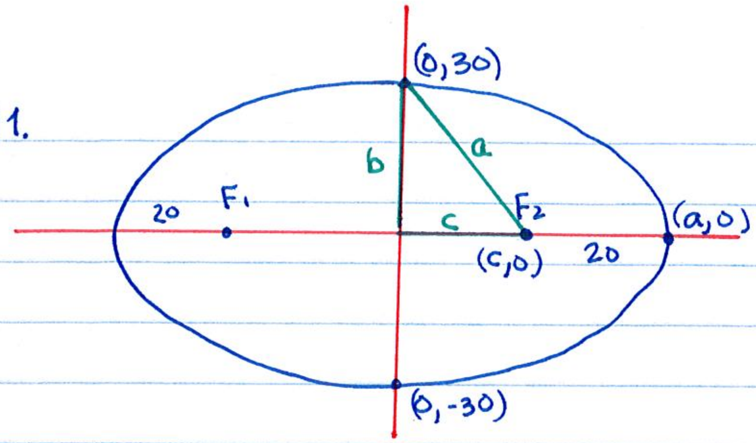
$\therefore m\angle BCO = 90° - 53.13°$

$= 36.87°$

Answer: Rhombus: $m\angle ABC = m\angle ADC = 2(53.13°) = \boxed{106.26°}$

$m\angle BCD = m\angle BAD = 2(36.87°) = \boxed{73.74°}$

11.



$b = 60 \div 2 = 30$

$c = a - 20$ or $a = c + 20$

$a^2 = b^2 + c^2$

$a^2 = (30)^2 + (a-20)^2$

$a^2 = 900 + a^2 - 40a + 400$

$a^2 = a^2 - 40a + 1300$

$0 = -40a + 1300$

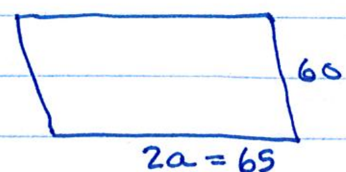
$40a = 1300$

$a = 32.5$

Rule

$$\frac{x^2}{(32.5)^2} + \frac{y^2}{30^2} = 1$$

Rectangle



Answer: Area of rectangle $= 65 \times 60$

$= 3\,900 \text{ cm}^2$

12. population 1: $a = 2000$

$c = 2$

$x = ?$

$y_1 = 2000(2)^x$

population 2: $a = 2\,048\,000$

$c = \frac{1}{2}$

$x = ?$

$y_2 = 2\,048\,000\left(\frac{1}{2}\right)^x$

$y_1 = y_2$

$2000(2)^x = 2\,048\,000\left(\frac{1}{2}\right)^x$

$2^x = 1024\left(\frac{1}{2}\right)^x$

$\left(\frac{a}{n}\right)^m = \frac{a^m}{n^m}$

$\frac{2^x}{\left(\frac{1}{2}\right)^x} = 1024$

$\left(2 \div \frac{1}{2}\right)^x = 1024$

$4^x = 1024$

12 (continued) $4^x = 1024$

$4^x = 4^5$

same base ⇒ same exponent

$\therefore x = 5$

$\log_4 1024 = x$

$\frac{\log 1024}{\log 4} = x$

$5 = x$

Answer : 5 days

13. $a = 200$

$c = 2$

$b = 2$ (2 doubling periods per year)

$y = 200(2)^{2x}$

let $y = 18\,500$

$18\,500 = 200(2)^{2x}$

$92.5 = 2^{2x}$

$\log_2 92.5 = 2x$

$\frac{\log 92.5}{\log 2} = 2x$

$6.531 = 2x$

$3.27 = x$

Answer: 3.27 years

14. $1400 = 1000\left(1 + \frac{t}{2}\right)^{2(3)}$

$1.4 = \left(1 + \frac{t}{2}\right)^6$

$\sqrt[6]{1.4} = 1 + t/2$

$1.05768 = 1 + t/2$

$0.5768 = t/2$

$0.11536 = t$

Answer : approximately 11.54%

15. $a = 25$

$b = 1/12$ (# of sublimation periods per hour)

$c = 1 - 0.09 = 0.91$

$y = 10$

$x = ?$

$$y = ac^{bx}$$

$$10 = 25(0.91)^{x/12}$$

$$0.4 = (0.91)^{x/12}$$

$$\log_{0.9} 0.4 = x/12$$

$$\frac{\log 0.4}{\log 0.91} = \frac{x}{12}$$

$$9.716 = \frac{x}{12}$$

$x = 116.59$ hours

Answer: 116.59 hours

16. $P(t) = 70\,000(2)^{t/50}$

$$100\,000 = 70\,000(2)^{t/50}$$

$$1.42857 = 2^{t/50}$$

$$\log_2 1.42857 = t/50$$

$$\frac{\log 1.42857}{\log 2} = t/50$$

$$0.514573 = t/50$$

$$25.73 = t$$

$2010 + 25.73 = 2035.73$

Answer: ~ 2035 or possibly 2036, depending when the survey took place in 2010.

17. ① $a = 28\,000$

$x = 4$

$c = 1 - 0.15 = .85$

$y = 28\,000(0.85)^4$

$y = 14\,616.18$

② $a = 14\,616.18$

$c = 1 - 0.1 = 0.9$

$y = 6\,632.13$

$x = ?$

$y = 14\,616.18(0.9)^x$

$6\,632.13 = 14\,616.18(0.9)^x$

$0.4537528 = 0.9^x$

$x = \log_{0.9}(0.4537528)$

$x = \dfrac{\log 0.4537528}{\log 0.9}$

$x = 7.5$

Anwer: After $(4 + 7.5) = 11.5$ years

18. $a = 17\,500$

$x = 3$

$y = 10\,000$

$c = ?$

$y = ac^x$

$10\,000 = 17500\, c^3$

$0.571429 = c^3$

$\sqrt[3]{0.571429} = c$

$0.83 = c$

$\therefore\ y = 17\,500(0.83)^x$ OR $y = 10000(0.83)^x$

let $y = 5000$

$5000 = 17\,500(0.83)^x$

$0.285714 = (0.83)^x$

$\log_{0.83} 0.285714 = x$

$\dfrac{\log 0.285714}{\log 0.83} = x$

$6.72 = x$

Answer: 6.7 years OR 6 years 9 months

$(0.72 \times 12) = 8.6$ months

19. $a = 5.5$

$c = 1 + 0.019 = 1.019$

$y = 9$

$x = ?$

$$y = ac^x$$
$$9 = 5.5(1.019)^x$$
$$1.\overline{63} = 1.019^x$$
$$\log_{1.019} 1.\overline{63} = x$$
$$\frac{\log 1.\overline{63}}{\log 1.019} = x$$
$$26.17 = x$$

Answer: $2010 + 26$

$= \boxed{2036}$

20. $a = 150$

$y = 75$ (½ of 150)

when $x = 10$ $y = 123$

$$y = ac^x$$
$$123 = 150(c)^{10}$$
$$0.82 = c^{10}$$
$$\sqrt[10]{0.82} = c$$
$$0.9804 = c$$

$$y = 150(0.9804)^x$$
$$75 = 150(0.9804)^x$$
$$0.5 = 0.9804^x$$
$$\log_{0.9804} 0.5 = x$$
$$\frac{\log 0.5}{\log 0.9804} = x$$
$$34.93 = x$$

Answer (to the nearest day): $\boxed{35 \text{ days}}$

21

$a = 1$

$y = 4/9$

$x = 2$

$c = ?$

$\frac{4}{9} = 1(c)^2$

$\frac{4}{9} = c^2$

$\sqrt{\frac{4}{9}} = c$

$\frac{2}{3} = c$

$a = 60$

$x = 12$

$\Rightarrow \quad y = 60\left(\frac{2}{3}\right)^{12}$

$y = 0.46$

Answer: 0.46 g

23.

$4x + 3y - 43 = 0$

$3y = -4x + 43$

$y = -4/3x + 43/3$

Tangent line

Radius: slope $= \frac{3}{4}$ (perpendicular to tangent line

Point $(3,2)$

Rule:

$y = \frac{3}{4}x + b$

$2 = \frac{3}{4}(3) + b$

$2 = 9/4 + b$

$-1/4 = b$

$y = \frac{3}{4}x - \frac{1}{4}$

Point where tangent line meets circle

by comparison

$\left(-\frac{4}{3}x + \frac{43}{3} = \frac{3}{4}x - \frac{1}{4}\right) \cdot 12$

$-16x + 172 = 9x - 3$

$175 = 25x$

$7 = x$

$y = \frac{3}{4}(7) - \frac{1}{4}$

$= \frac{21}{4} - \frac{1}{4}$

$= \frac{20}{4} = 5$

$(7,5)$

23 continued

Equation of the circle

$$(x-3)^2 + (y-2)^2 = r^2$$

using (7,5)

$$(7-3)^2 + (5-2)^2 = r^2$$

$$4^2 + 3^2 = r^2$$

$$25 = r^2$$

Answer: $(x-3)^2 + (y-2)^2 = 25$

27. $\log_2(x^2+5) - \log_2 5 = \log_2 6$

$x^2+5>0$

$x^2>-5$

$x \in \mathbb{R}$

$$\log_2(x^2+5) = \log_2 6 + \log_2 5$$

$$\log_2(x^2+5) = \log_2(6\times5)$$

$$\log_2(x^2+5) = \log_2 30$$

matching logarithms; $\therefore$ $x^2+5 = 30$

$$x^2 = 25$$

$$x = \pm 5$$

Answer: $x = \{-5, 5\}$

28. A: $a(12) = 1000\log_4(12)$

$= 1000\dfrac{\log 12}{\log 4}$

$= 1000(1.79248)$

$= \$1792.48$

Profit $= \$1792.48 - \$800 = \$992.48$

B: $b(12) = 1000\log_5(12)$

$= 1000\dfrac{\log 12}{\log 5}$

$= 1000(1.543959)$

$= \$1543.96$

Profit $= \$1543.96 - \$500 = \$1043.96$

Answer: Company B

29. $\log_5(x-1) + \log_5(x+3) - 1 = 0$

$x-1>0 \qquad x+3>0$

$x>1 \qquad x>-3$

$\log_5[(x-1)(x+3)] - 1 = 0$

$\log_5(x^2+2x-3) = 1$

$x^2+2x-3 = 5$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0$ or $x-2=0$

$x=\{-4, 2\}$

reject $x=-4$

Answer: $x=2$

30.

$x \geq 0$

$y \geq 0$

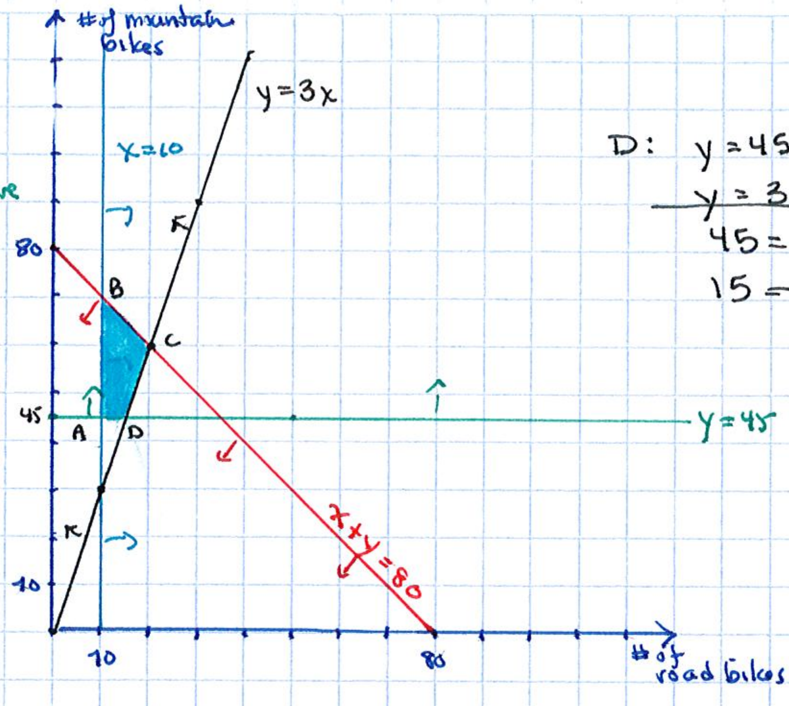
solid, shade below $\quad x + y \leq 80$

$y \geq 45$ solid above

$x \geq 10$ solid righ

$y \geq 3x$ solid above

$P = 250x + 175y$



D: $\quad y = 45$

$\quad y = 3x$

$\quad 45 = 3x$

$\quad 15 = x$

Vertices	$P = 250x + 175y$
A(10, 45)	\$ 10 375
B(10, 70)	\$ 14 750
C (20, 60)	\$ 15 500
D(15, 45)	\$ 11 625

Answer: \$ 15 500

31 $\quad$ x: # of crabs

y: # of lobster

$x \geq 0$

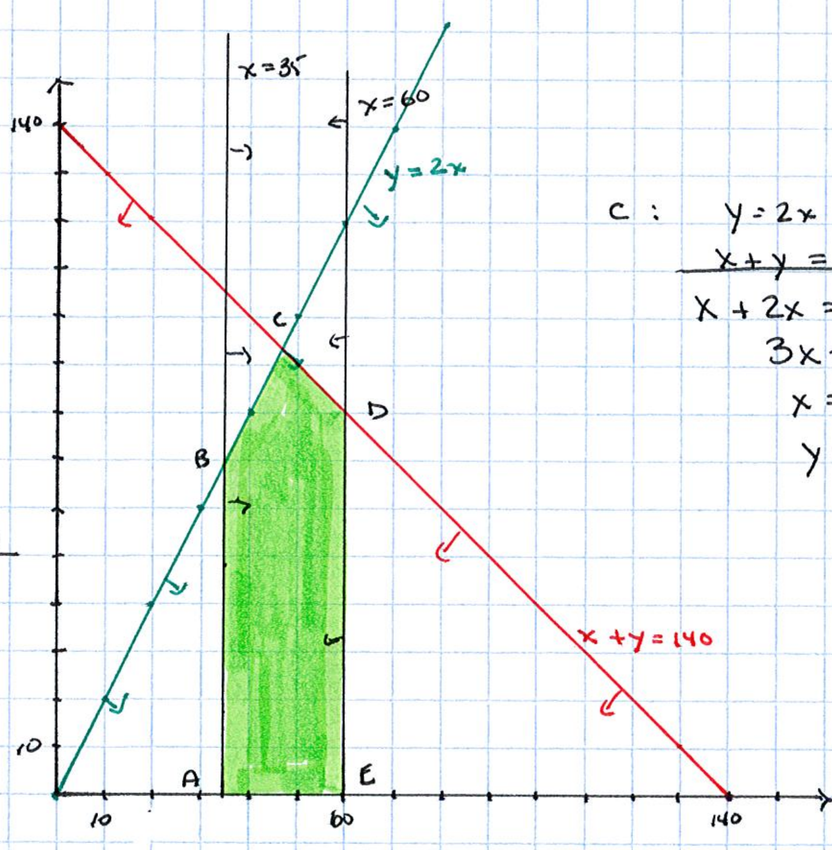
$y \geq 0$

$x \geq 35$

$x \leq 60$

$y \leq 2x$

$x + y \leq 140$



C : $\quad y = 2x$

$\quad x + y = 140$

$x + 2x = 140$

$3x = 140$

$x = \frac{140}{3}$

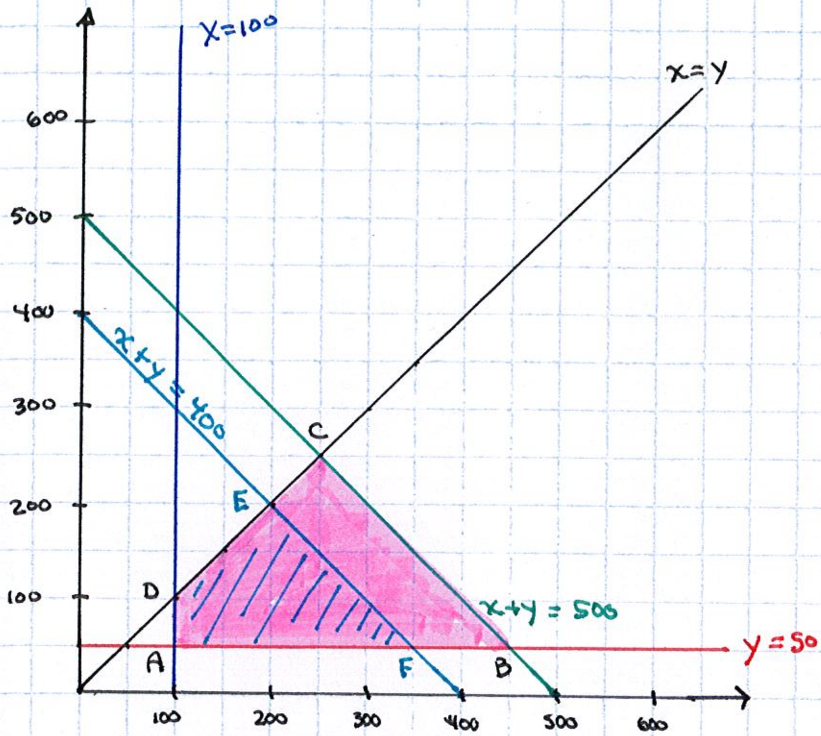
$y = \frac{280}{3}$

Vertices	$R = 9.60x + 8.70y$
(35, 0)	\$ 336
(35, 70)	\$ 945
$(\frac{140}{3}, \frac{280}{3})$	\$ 1260 *
D(60, 80)	\$ 1272
(60, 0)	\$ 576

* not useable

Answer: \$ 1272

33.



A(100,50)
B(450,50)
C(250,250)
D(100,100)

E(200,200)
F(350,50)

Before

Vertices	R = 1x + 1.50y
A (100,50)	\$175
B (450,50)	\$525
C (250,250)	\$625
D (100,100)	\$250

maximum possible Revenue = \$625

New constraint: $x + y \leq 400$

After

Vertices	R = 1x + 1.5y
A (100,50)	\$175
D (100,100)	\$250
E (200,200)	\$500
F (350,50)	\$425

maximum possible Revenue = \$500

Decrease = \$625 − \$500 = \$125

Answer: \$125

34. Semi-ellipse $a = 15$ $c = 12$

$$a^2 = b^2 + c^2$$
$$b^2 = a^2 - c^2$$
$$b^2 = 225 - 144$$
$$b^2 = 81$$
$$b = \pm 9$$

$\therefore$ Height of semi-ellipse $= 9\text{cm}$

Circle: $r = 5$ centre $(0, 9+5)$
$$(0, 14)$$

Parabola: vertex: $(0, 9+10) = (0, 19)$

directrix : $y = 14$

$$c = 19 - 14 = 5$$

Rule : $(x-h)^2 = 4c(y-k)$
$$x^2 = 4(5)(y-19)$$
$$x^2 = 20(y-19)$$

let $x = \frac{36}{2} = 18$
$$18^2 = 20(y-19)$$
$$324 = 20(y-19)$$
$$16.2 = y - 19$$
$$35.2 = y$$

Answer: The height is 35.2 cm

35. Directrix : Route 173 $c = 200 \div 2 = 100$

Restaurant : Focus

let the vertex be $(0,0)$
$$x^2 = -4cy$$
$$x^2 = -400y$$

let $x = 800 \div 2 = 400$
$$(400)^2 = -400y$$
$$1600 = -400y$$
$$-400 = y$$

Answer : 400 m

36. parabolas : $x^2 - 8x - y + 16 = 0$

$$x^2 - 8x = y - 16$$

$-8 \div 2 = -4$

$(-4)^2 = 16$

$$x^2 - 8x + 16 = y - 16 + 16$$

$$(x-4)^2 = y \qquad \text{vertex } (4,0)$$

$$x^2 - 8x = -y - 8$$

$$x^2 - 8x + 16 = -y - 8 + 16$$

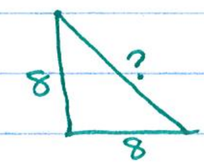
$$(x-4)^2 = -y + 8$$

$$(x-4)^2 = -1(y-8) \qquad \text{vertex} = (4,8)$$

$\therefore$ diameter of circle = 8cm centre = $(4,4)$

rule: $(x-4)^2 + (y-4)^2 = 16$

square 8cm by 8cm



$$c^2 = 8^2 + 8^2$$

$$c^2 = 64 + 64$$

$$c^2 = 128$$

$$c = \sqrt{128} = \sqrt{64 \cdot 2} = 8\sqrt{2} \text{ or } 11.31$$

Answer: $8\sqrt{2}$ cm or 11.31cm

37. circle : $x^2 + y^2 - 32x - 10y + 279 = 0$

$$x^2 - 32x + y^2 - 10y = -279$$

$-32 \div 2 = -16$ $\quad$ $-10 \div 2 = -5$

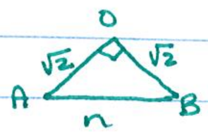
$(-16)^2 = 256$ $\quad$ $(-5)^2 = 25$

$$(x^2 - 32x + 256) + (y^2 - 10y + 25) = -279 + 256 + 25$$

$$(x-16)^2 + (y-5)^2 = 2$$

37 continued

parabola: vertex $(16,5)$



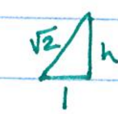
$n^2 = (\sqrt{2})^2 + (\sqrt{2})^2$

$n^2 = 2+2$

$n^2 = 4$

$n = 2$

$A(15, y)$ $\quad B(17, y)$



$h^2 + 1^2 = (\sqrt{2})^2$

$h^2 + 1 = 2$

$h^2 = 1$

$h = 1$

$\therefore A(15,4) \quad B(17,4)$

Rule: $(x-16)^2 = 4c(y-5)$

$(17-16)^2 = 4c(4-5)$

$1^2 = 4c(-1)$

$1 = -4c$

$-1/4 = c$ $\quad \therefore (x-16)^2 = -(y-5)$

let $y=0$

$(x-16)^2 = -(-5)$

$(x-16)^2 = 5$

$x - 16 = \pm\sqrt{5}$

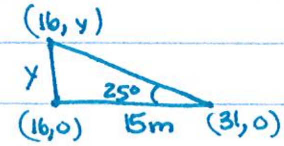
$x = 16 \pm \sqrt{5}$

$C(16-\sqrt{5}, 0) \quad D(16+\sqrt{5}, 0)$

Answer: $d(C,D) = (16+\sqrt{5}) - (16-\sqrt{5})$

$= \boxed{2\sqrt{5}\,m \text{ or } 4.47\,m}$

38. square root function : $V(0, 15)$ $P(16, y)$



$$\tan 25° = \frac{y}{15}$$

$$y = 15 \cdot \tan 25°$$

$$y = 6.99\text{ m}$$

$$\therefore P(16, 6.99)$$

Rule: $y = a\sqrt{x-h} + k$

$$6.99 = a\sqrt{16} + 15$$

$$-8.01 = 4a$$

$$-2 = a$$

$$y = -2\sqrt{x} + 15$$

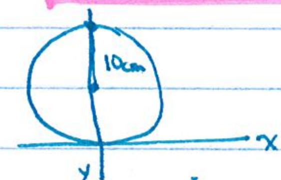
let $x = 8$

$$y = -2\sqrt{8} + 15$$

$$y = 9.34$$

Answer: 9.34 m

39. x-axis : bottom of clock

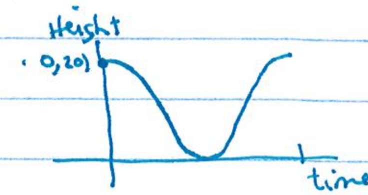


starting position : (0,20)

period = 60 seconds

max = 20 , min = 0

$A = \frac{20}{2} = 10 \quad \therefore a = \pm 10$



$b = \pm\left(\frac{2\pi}{60}\right) = \pm \frac{\pi}{30}$

$k = 20 - 10 = 10$

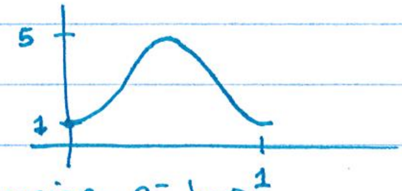
starting at max : cosine function , $h = 0$, a^+

Answer: $f(x) = 10\cos\left(\frac{\pi}{30}x\right) + 10$

40. min = 1

max = 5

p = 1 minute



starting position : minimum ⇒ cosine, a^-, h=0

$A = \frac{5-1}{2} = \frac{4}{2} = 2 \qquad a = \pm 2$

$b = \pm \frac{2\pi}{1} = \pm 2\pi$

$k = 5 - 2 = 3$

Rule: $f(x) = -2\cos(2\pi x) + 3$

$x = 12 + \frac{40}{60} = 12.\overline{6}$ minutes

$f(12.\overline{6}) = -2\cos(25.\overline{3}\pi) + 3$

$= -2(-0.5) + 3$

$= 1 + 3$

$= 4$

Answer: 4 metres

41. r = 10m

min = 2

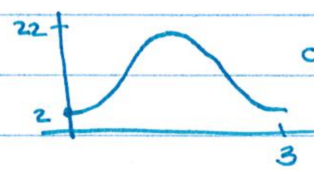
max = 2 + 20 = 22 m

p = 3 minutes

$A = \frac{22-2}{2} = 10 \therefore a = \pm 10$

$k = 22 - 10 = 12$

$b = \pm \frac{2\pi}{3}$



cosine function
a^-, h 0

$f(x) = -10\cos\left(\frac{2\pi}{3}x\right) + 12$

let x = 7

$f(7) = -10\cos\left(\frac{14\pi}{3}\right) + 12$

$= -10(-0.5) + 12$

$= 5 + 12$

$= 17$

Answer : 17 m

42. 1^{ST}: $v(0,3)$ $P_1(4,1)$ $b = +1$

$$y = a\sqrt{x-h} + k$$
$$1 = a\sqrt{4} + 3$$
$$1 = 2a + 3$$
$$-2 = 2a$$
$$-1 = a \qquad \underline{y = -\sqrt{x} + 3}$$

2^{ND}: $v(2,0)$ $P_2(6,2)$ $b = +1$

$$y = a\sqrt{x-h} + k$$
$$2 = a\sqrt{6-2}$$
$$2 = a\sqrt{4}$$
$$2 = 2a$$
$$1 = a \qquad \underline{y = \sqrt{x-2}}$$

$$\sqrt{x-2} = -\sqrt{x} + 3$$
$$(\sqrt{x-2})^2 = (-\sqrt{x} + 3)^2$$
$$x - 2 = x - 6\sqrt{x} + 9$$
$$0 = -6\sqrt{x} + 11$$
$$-11 = -6\sqrt{x}$$
$$11/6 = \sqrt{x}$$
$$3.36\bar{1} = 121/36 = x$$

Answer: After $3.36 - 2 = \underline{\underline{1.36\,s}}$

44. Absolute Value: $P(0,2)$ $\quad v(1,10)$

$y = a|x-h| + k$

$2 = a|0-1| + 10$

$2 = a|-1| + 10$

$-8 = a$

$y = -8|x-1| + 10$

let $y = 1$

$1 = -8|x-1| + 10$

$-9 = -8|x-1|$

$1.125 = |x-1|$

$\pm 1.125 = x-1$

$\{-0.125, 2.125\} = x$

$x = 2.125$ s.

Semi-Parabola: $v(2.125, 1)$ $\quad P(3.125, 3)$

① $(y-k)^2 = 4c(x-h)$

$(y-1)^2 = 4c(x-2.125)$

$(3-1)^2 = 4c(3.125-2.125)$

$2^2 = 4c(1)$

$4 = 4c$

$1 = c$

$(y-1)^2 = 4(x-2.125)$

let $y = 5$

$(5-1)^2 = 4(x-2.125)$

$4^2 = 4(x-2.125)$

$4 = x-2.125$

$6.125 = x$

② square root function

$y = a\sqrt{x-h} + k$

$3 = a\sqrt{3.125-2.125} + 1$

$3 = a\sqrt{1} + 1$

$2 = a$

$y = 2\sqrt{x-2.125} + 1$

let $y = 5$

$5 = 2\sqrt{x-2.125} + 1$

$4 = 2\sqrt{x-2.125}$

$2 = \sqrt{x-2.125}$

$4 = x-2.125$

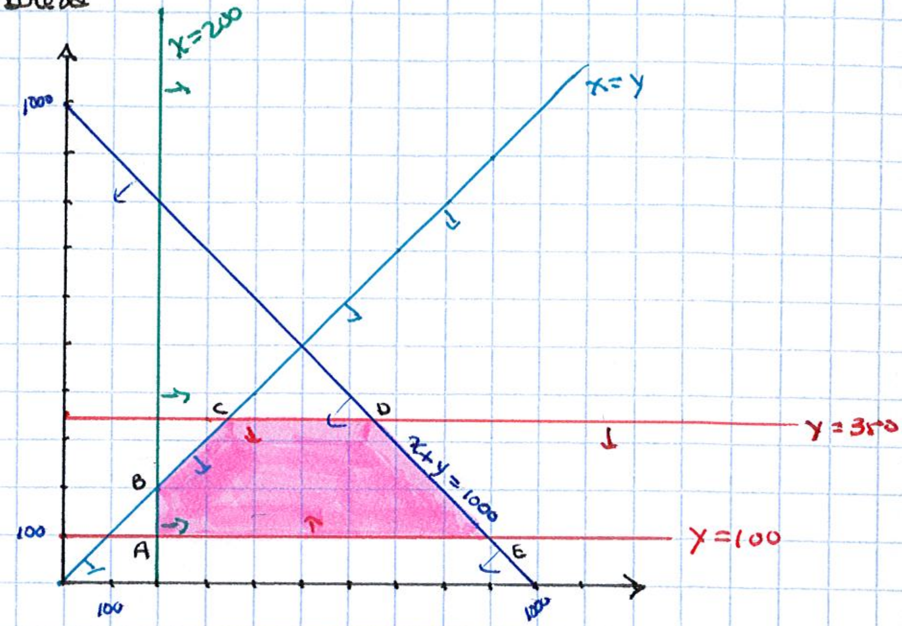
$6.125 = x$

Answer: 6.125 seconds

45 x: # of loaves of raisin bread
y: # of loaves of olive bread

$x \geq 0$
$y \geq 0$
$x + y \leq 1000$
$x \geq 200$
$y \geq 100$
$y \leq 350$
$x \geq y$

$P = 0.10x + 0.20y$



Vertices	$P = 0.10x + 0.20y$	
A(200,100)	\$40	
B(200, 200)	\$60	
C(350, 350)	\$105	
D(650, 350)	\$135	maximum profit
E(900, 100)	\$110	

Answer: 650 raisin bread loaves and 350 olive bread loaves.

46. $\csc A(\csc A + \cot A) = \frac{1}{1-\cos A}$

$$\csc^2 A + \csc A \cot A = \frac{1}{1-\cos A}$$

$$\frac{1}{\sin^2 A} + \frac{1}{\sin A}\cdot\frac{\cos A}{\sin A} = \frac{1}{1-\cos A}$$

$$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$

$$\frac{1+\cos A}{1-\cos^2 A} = \frac{1}{1-\cos A}$$

$$\frac{(\cancel{1+\cos A})}{(\cancel{1+\cos A})(1-\cos A)} = \frac{1}{1-\cos A}$$

$$\frac{1}{1-\cos A} = \frac{1}{1-\cos A}$$

47. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

$$\left(\frac{1+\sin\theta}{1+\sin\theta}\right)\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right) = 2\sec\theta$$

$$\frac{1+2\sin\theta+\sin^2\theta}{(1+\sin\theta)\cos\theta} + \frac{\cos^2\theta}{(1+\sin\theta)\cos\theta} = 2\sec\theta$$

$$\frac{1+2\sin\theta+(\sin^2\theta+\cos^2\theta)}{(1+\sin\theta)(\cos\theta)} = 2\sec\theta$$

$$\frac{1+2\sin\theta+1}{(1+\sin\theta)(\cos\theta)} = 2\sec\theta$$

$$\frac{2+2\sin\theta}{(1+\sin\theta)(\cos\theta)} = 2\sec\theta$$

$$\frac{2\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}(\cos\theta)} = 2\sec\theta$$

$$2\cdot\left(\frac{1}{\cos\theta}\right) = 2\sec\theta$$

$$2\sec\theta = 2\sec\theta$$

48. $$\frac{\sec\theta}{1-\cos\theta} - \frac{\sec\theta}{1+\cos\theta} = 2\csc^2\theta$$

$$\left(\frac{1+\cos\theta}{1+\cos\theta}\right)\frac{\sec\theta}{1-\cos\theta} - \frac{\sec\theta}{1+\cos\theta}\left(\frac{1-\cos\theta}{1-\cos\theta}\right) = 2\csc^2\theta$$

$$\left(\frac{\sec\theta + \sec\theta\cos\theta}{1-\cos^2\theta}\right) - \left(\frac{\sec\theta - \sec\theta\cos\theta}{1-\cos^2\theta}\right) = 2\csc^2\theta$$

$$\left(\frac{\sec\theta + 1}{\sin^2\theta}\right) - \left(\frac{\sec\theta - 1}{\sin^2\theta}\right) = 2\csc^2\theta$$

$$\frac{2}{\sin^2\theta} = 2\csc^2\theta$$

$$2\left(\frac{1}{\sin^2\theta}\right) = 2\csc^2\theta$$

$$2\csc^2\theta = 2\csc^2\theta$$

49. $$\frac{2}{\csc a} + 2\sin a\tan^2 a = \frac{2\tan a}{\cos a}$$

$$2\left(\frac{1}{\csc a}\right) + 2\sin a(\sec^2 a - 1) = \frac{2\tan a}{\cos a}$$

$$2\cancel{\sin a} + 2\sin a\sec^2 a - 2\cancel{\sin a} = \frac{2\tan a}{\cos a}$$

$$2\sin a\sec^2 a = \frac{2\tan a}{\cos a}$$

$$2\sin a \cdot \frac{1}{\cos^2 a} = \frac{2\tan a}{\cos a}$$

$$\frac{2\sin a}{\cos a \cdot \cos a} = \frac{2\tan a}{\cos a}$$

$$\frac{2\tan a}{\cos a} = \frac{2\tan a}{\cos a}$$

50. $$\frac{2 - 2\sin A\cos A\tan A}{2} = \cos^2 A$$

$$\frac{2 - 2\sin A\cancel{\cos A}\frac{\sin A}{\cancel{\cos A}}}{2} = \cos^2 A$$

$$\frac{2 - 2\sin^2 A}{2} = \cos^2 A$$

$$\frac{\cancel{2}(1-\sin^2 A)}{\cancel{2}} = \cos^2 A$$

$$1 - \sin^2 A = \cos^2 A$$

$$\cos^2 A = \cos^2 A$$

51. $$\frac{\csc A - \sin A}{\cos A} = \cot A$$

$$\frac{\frac{1}{\sin A} - \sin A}{\cos A} = \cot A$$

$$\frac{\frac{1}{\sin A} - \frac{\sin^2 A}{\sin A}}{\cos A} = \cot A$$

$$\frac{\frac{1-\sin^2 A}{\sin A}}{\cos A} = \cot A$$

$$\frac{\frac{\cos^2 A}{\sin A}}{\cos A} = \cot A$$

$$\frac{\cos^{\cancel{2}} A}{\sin A} \cdot \frac{1}{\cancel{\cos A}} = \cot A$$

$$\frac{\cos A}{\sin A} = \cot A$$

$$\cot A = \cot A$$

59. a) $\sin^2 x + \cos x = 1$

$1 - \cos^2 x + \cos x = 1$

$\cos x - \cos^2 x = 0$

$\cos x(1 - \cos x) = 0$

$\cos x = 0 \qquad 1 - \cos x = 0$

$x = \{\frac{\pi}{2}, \frac{3\pi}{2}\} \qquad 1 = \cos x$

$x = 0$

Answer: $x = \{0, \frac{\pi}{2}, \frac{3\pi}{2}\}$

b) $2\sin^2 x - \cos x - 2 = 0$

$2(1 - \cos^2 x) - \cos x - 2 = 0$

$\cancel{2} - 2\cos^2 x - \cos x - \cancel{2} = 0$

$-2\cos^2 x - \cos x = 0$

$\cos x(-2\cos x - 1) = 0$

$\cos x = 0 \qquad -2\cos x - 1 = 0$

$-2\cos x = 1$

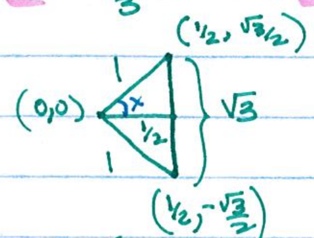
$\cos x = -\frac{1}{2}$

$x = \{\frac{\pi}{2}, \frac{3\pi}{2}\} \qquad x = \{\frac{2\pi}{3}, \frac{4\pi}{3}\}$

Answer: $x = \{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\}$

60. $\sin x = \frac{\sqrt{3}}{2}$ $\qquad x = \frac{\pi}{3}$ rad or $60°$

$\therefore \cos x = \frac{1}{2}$



Area of $\Delta AOB = \frac{\sqrt{3} \cdot \frac{1}{2}}{2} = \frac{\sqrt{3}}{4} u^2$

Area of sector

$= \frac{1}{3}\pi r^2 = \frac{1}{3}\pi(1) = \frac{\pi}{3}$

$\frac{120°}{360°} = \frac{1}{3}$

Answer: Area shaded region $= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) u^2$ or $0.614\ u^2$

61. $2\cos^2 x - 3\sin x - 3 = 0$

$2(1-\sin^2 x) - 3\sin x - 3 = 0$

$2 - 2\sin^2 x - 3\sin x - 3 = 0$

$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$ a trinomial of the form

$(\sin x + 1)(2\sin x + 1) = 0$ $2x^2 + 3x + 1 = 0$

$ac = 2$, $b = 3$ } 2, 1

$\sin x + 1 = 0$ or $2\sin x + 1 = 0$

$\sin x = -1$ $\quad$ $2\sin x = -1$ $\quad$ $2x^2 + 2x + x + 1 = 0$

$x = \frac{3\pi}{2}$ $\quad$ $\sin x = -1/2$ $\quad$ $2x(x+1) + 1(x+1) = 0$

$x = \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ $\quad$ $(x+1)(2x+1) = 0$

$x = \left\{\cancel{\frac{7\pi}{6}}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$ $\quad$ $x \in \left[\frac{3\pi}{2}, 2\pi\right]$

Answer: $x = \left\{\frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

63. $2\sin^2 x - 4\cos^2 x + 1 = 0$

$2(1-\cos^2 x) - 4\cos^2 x + 1 = 0$

$2 - 2\cos^2 x - 4\cos^2 x + 1 = 0$

$3 - 6\cos^2 x = 0$

$3(1 - 2\cos^2 x) = 0$

$1 - 2\cos^2 x = 0$

$1 = 2\cos^2 x$

$1/2 = \cos^2 x$

$\pm\frac{\sqrt{2}}{2} = \pm 1/\sqrt{2} = \pm\sqrt{\frac{1}{2}} = \cos x$

$\cos x = \pm \sqrt{2}/2$ $\quad$ $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x = \left\{\frac{\pi}{4}, \frac{7\pi}{4}\right\}$

$\frac{7\pi}{4} \Rightarrow -\frac{\pi}{4}$

Answer: $x = \left\{-\frac{\pi}{4}, \frac{\pi}{4}\right\}$

64. $\frac{x^2}{25} - \frac{y^2}{144} = 1$

$a = 5 \quad b = 12$

$c^2 = a^2 + b^2$

$c^2 = 25 + 144$

$c^2 = 169$

$c = 13 \quad F_1(13,0) \quad F_2(-13,0)$

let $x = 13$

$\frac{169}{25} - \frac{y^2}{144} = 1$

$6.76 - \frac{y^2}{144} = 1$

$-\frac{y^2}{144} = -5.76$

$y^2 = (-5.76)(-144)$

$y^2 = 829.44$

$y = \pm 28.8$

max $= 28.8$

min $= -28.8$

$a = \pm 28.8$

$p = 13 + 13 = 26$

$b = \pm \frac{2\pi}{26} = \pm \frac{\pi}{13}$

Point $(0, -28.8)$

cosine function: min, $h=0$, a-

Answer: $y = -28.8\cos\left(\frac{\pi}{13}x\right)$

* other answers are possible

65. $\log_4(x^2+15x)=2$

$4^2 = x^2+15x$

$16 = x^2+15x$

$0 = x^2+15x-16$

$0 = (x+16)(x-1)$

$x+16=0$ or $x-1=0$

$x=-16$ $\quad$ $x=1$

Test

$\log_4\left((-16)^2+15(-16)\right)$ $\quad$ $\log_4\left(1^2+15(1)\right)$

$\log_4(256+-240)$ $\quad$ $\log_4(16)$ ✓

$\log_4(16)$ ✓

Answer: $x=\{-16, 1\}$

66. $\log_2(x-3)+\log_2(2x)=3$ $\quad$ $x-3>0$ $\quad$ $2x>0$

$\log_2(2x(x-3))=3$ $\quad$ $x>3$ $\quad$ $x>0$

$\log_2(2x^2-6x)=3$

$2x^2-6x=2^3$

$2x^2-6x=8$

$2x^2-6x-8=0$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x-4=0$ or $x+1=0$

$x=4$ $\quad$ $x=-1$

Reject

Answer: $x=4$

67. $3\log_4 2 + \log_2 \sqrt{8} + 2\log_{1/4} 16 + \log_2 4^2$

$\log_4 2^3 + \log_2 8^{1/2} + \log_{1/4} 16^2 + \log_2 16$

$\log_4 8 + \frac{1}{2}\log_2 8 + \log_{1/4} 256 + \log_2 16$

$1.5 + \frac{1}{2}(3) + (-4) + 4$

$1.5 + 1.5 - 4 + 4$

3

Answer: A

68. $\log_2 (x+3) + \log_2 (2x+4) - \log_{11} 121 = 0$

$\log_2 [(x+3)(2x+4)] - 2 = 0$

$\log_2 (2x^2 + 10x + 12) = 2$

$2x^2 + 10x + 12 = 2^2$

$2x^2 + 10x + 12 = 4$

$2x^2 + 10x + 8 = 0$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x+4=0$ or $x+1=0$

$x=-4$ or $x=-1$

Reject

$x+3>0$ || $2x+4>0$

$x>-3$ || $2x>-4$

$x>-2$

$\therefore\ x=-1$

Answer: B

69. $\frac{1}{2}(\log_a m^4 + 2\log_a p) - 5\log_a a$

$\frac{1}{2}(4\log_a m + 2\log_a p) - 5$

$2\log_a m + \log_a p - 5$

$\log_a m^2 + \log_a p - 5$

$\log_a (m^2 p) - 5$

Answer: B

70. $2\log_b 1 + \log_b b^2 + \log_b\left(\frac{1}{b}\right)$

$2(0) + 2\log_b b + \log_b(b^{-1})$

$0 + 2(1) + -1\log_b b$

$0 + 2 + -1(1)$

$0 + 2 - 1$

1

$\boxed{\text{Answer: B}}$

71. $\dfrac{\log_c 20c}{\log_c 50 - \log_c 5} = \dfrac{\log_c 20c}{\log_c\left(\frac{50}{5}\right)}$

$= \dfrac{\log_c 20c}{\log_c 10}$ ⇒ change of base law

$\log_m a = \dfrac{\log_c a}{\log_c m}$

$= \log_{10} 20c$ ← $m = 10$ $a = 20c$

$= \log 20c$

$\boxed{\text{Answer: C}}$

72. $\log_2 \sqrt{8} + \log_3 \frac{1}{3} + 6^{\log_6 1}$

$\log_2 8^{1/2} + \log_3 3^{-1} + 6^0$

$\frac{1}{2}\log_2 8 - 1\log_3 3 + 1$

$\frac{1}{2}(3) - 1(1) + 1$

$\frac{3}{2} - 1 + 1$

$\frac{3}{2}$

$\boxed{\text{Answer: C}}$

73. $\log_a\left(\frac{x^2 a}{y}\right) = \log_a x^2 + \log_a a - \log_a y$

$= 2\log_a x + 1 - \log_a y$

$= 2(5) + 1 - 7$

$= 10 + 1 - 7$

$= 4$

Answer : B

74. $3(\log a^2 - \log 2a) + 4\log a$

$3\left(\log \frac{a^2}{2a}\right) + \log a^4$

$3\log\left(\frac{a}{2}\right) + \log a^4$

$\log\left(\frac{a}{2}\right)^3 + \log a^4$

$\log \frac{a^3}{8} + \log a^4$

$\log\left(\frac{a^3}{8} \cdot a^4\right)$

$= \log\left(\frac{a^7}{8}\right)$

Answer: B

75. $\log_a m + \log_a n - 3.\log_a n$

$\log_a m + \log_a n - \log_a n^3$

$\log_a\left(\frac{m \cdot \cancel{n}}{n^{\cancel{3}\,2}}\right)$

$\log_a\left(\frac{m}{n^2}\right)$

Answer: B