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1. Determine the following absolute values.

a) $|+8| =$ ***8***

b) $|-4.7| =$ ***4.7***

c) $|0| =$ ***0***

d) $|\pi| =$ π

e) $|-6.53| =$ ***6.53***

f) $\left|+\frac{3}{4}\right| =$ $\frac{3}{4}$

g) $\left|-\frac{2}{3}\right| =$ $\frac{2}{3}$

h) $\left|-\frac{5}{18}\right| =$ $\frac{5}{18}$

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2. Complete the following using the appropriate symbol =, >, < .

a) $|x + 5|$ $\geq$ 0

b) $|x - 3|$ $=$ $|3 - x|$

c) $|2(x - 1)|$ $=$ $2|x - 1|$

d) $|7 - 12|$ $>$ $|7| - |12|$

e) $\left|\frac{x + 2}{x - 1}\right|$ $=$ $\frac{|x + 2|}{|x - 1|}$

f) $|-6 + 9|$ $<$ $|-6| + |9|$

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3. Solve the following equations.

a) $|x| = 12$

$S = \{-12, 12\}$

b) $|x| = -8$

$S = \varnothing$

c) $|x + 5| = 0$

$S = \{-5\}$

d) $|2x + 1| = 7$

$S = \{-4, 3\}$

e) $\left|\frac{1}{2}x - 5\right| = 4$

$S = \{2, 18\}$

f) $|6 - x| = -3$

$S = \varnothing$

4. Solve the following equations.

a) $2|x - 5| - 4 = 0$

$S = \{3, 7\}$

b) $-2|3x - 1| + 4 = -6$

$S = \left\{-\frac{4}{3}, 2\right\}$

c) $12 - |6 - 2x| = 3$

$S = \left\{-\frac{3}{2}, \frac{15}{2}\right\}$

d) $|x - 5| + 8 = 2$

$S = \varnothing$

e) $-3|2x + 5| + 6 = 6$

$S = \left\{-\frac{5}{2}\right\}$

f) $|4x - 5| + 6 = 9$

$S = \left\{\frac{1}{2}, 2\right\}$

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10. Write the rules of the following functions in the form $y = a|x - h| + k$ and identify the parameters a, h and k.

a) $y = -2|3x + 3| + 5$

$y = -6|x + 1| + 5$; $a = -6$, $h = -1$, $k = 5$

b) $y = 4|6 - 3x| + 5$

$y = 12|x - 2| + 5$; $a = 12$, $h = 2$, $k = 5$

c) $y = -\frac{1}{2}|8x - 4| + 3$

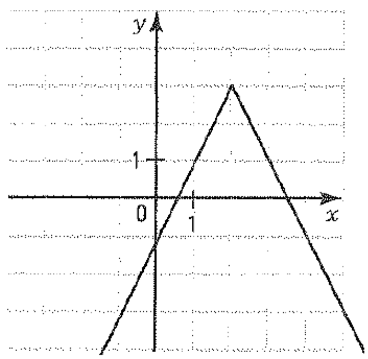
$y = -4\left|x - \frac{1}{2}\right| + 3$; $a = -4$, $h = \frac{1}{2}$, $k = 3$

d) $y = -\frac{5}{6}\left|4 - \frac{1}{5}x\right| + 3$

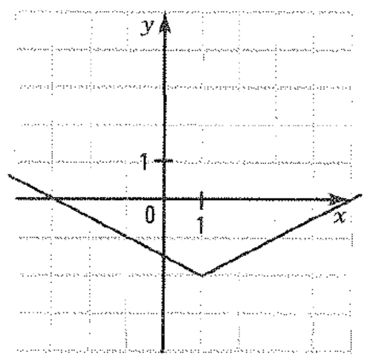
$y = -\frac{1}{6}|x - 20| + 3$; $a = -\frac{1}{6}$, $h = 20$, $k = 3$

11. Graph the following functions.

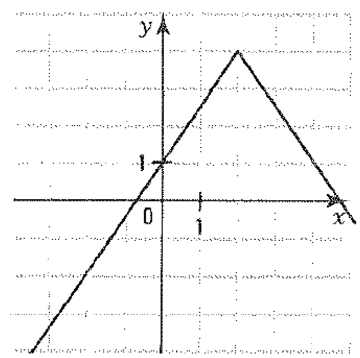
a) $y = -2|x - 2| + 3$



b) $y = \frac{1}{8}|4 - 4x| - 2$



c) $y = -\frac{1}{2}|3x - 6| + 4$



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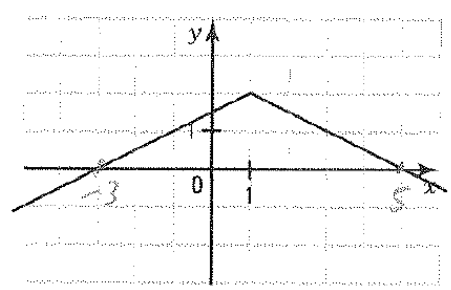
12. Represent the graph and do a study of the function $f(x) = -\frac{1}{4}|2(x - 1)| + 2$.



dom = ℝ; ran = $]-\infty, 2]$.

Zeros: −3 and 5.

Initial value: 1.5.

Sign: $f(x) \geq 0$ over $[-3, 5]$.

$f(x) < 0$ over $]-\infty, 2[\cup]5, +\infty[$.

Variation: $f \nearrow$ over $]-\infty, 1]$; $f \searrow$ over $[1, +\infty[$

Extrema: max = 2

13. Determine the domain and range of each of the following functions.

a) $y = -2|x + 5| - 1$

dom = ℝ, ran =]-∞, -1]

b) $y = \frac{1}{4}|-2(x - 1)| + 5$

dom = ℝ, ran = [5, +∞[

14. Determine the zeros of the following functions.

a) $y = 3|x - 5| - 6$ *3 and 7*

b) $y = -\frac{1}{2}|6 - 3x| + 4$ $-\frac{2}{3}$ *and* $\frac{14}{3}$

c) $y = 4|2x + 1| + 8$ *No zero*

d) $y = -5|6 - x|$ *6*

15. Consider the linear function $f(x) = 2x - 3$ and the absolute value function $g(x) = 3|3x + 5| - 4$. Determine the initial value of the composite $g(2x-3) = 3|6x-4|-4$

a) $g \circ f(0)$ *8*

b) $f \circ g(0)$ *19* $f(11) = 19$

16. Determine the interval over which each of the following functions is positive.

a) $y = -\frac{1}{3}|x - 5| + 2$ 5 ± 6

f(x) ≥ 0 over [-1, 11]

b) $y = 2|3 - 2x| - 4$

f(x) ≥ 0 over $\left]-\infty, \frac{1}{2}\right] \cup \left[\frac{5}{2}, +\infty\right[$

c) $y = \frac{3}{4}|-2x + 4| - 3$

f(x) ≥ 0 over]-∞, 0] ∪ [4, +∞[

d) $y = 3|x - 5| + 6$

f(x) ≥ 0 over ℝ

17. Determine the interval over which each of the following functions is increasing.

a) $y = 5|6 - 4x| + 2$

f ↗ over $\left[\frac{3}{2}, +\infty\right[$

b) $y = -3|2x + 4| + 5$

f ↗ over]-∞, -2]

18. Determine the solution set to each of the following inequalities.

a) $|x - 5| > 3$

S =]-∞, 2[∪]8, +∞[

b) $|6 - x| \leq 1$

S = [5, 7]

c) $|3x - 2| \geq 4$

S = $\left]-\infty, -\frac{2}{3}\right] \cup [2, +\infty[$

d) $|2x + 5| \leq 0$

S = $\left\{-\frac{5}{2}\right\}$

e) $-2|x + 1| + 5 > -5$

S =]-6, 4[

f) $3|2 - x| + 4 > 1$

S = ℝ

g) $6 - 3|x - 1| \leq 0$

S =]-∞, -1] ∪ [3, +∞[

h) $-|2x - 1| + 5 > 0$

S =]-2, 3[

i) $\left|\frac{x}{2} - 1\right| > 0$

S = ℝ\{2}

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19. Study each of the following functions and complete the following table.

	$f(x) = -2\|x - 1\| + 4$	$f(x) = 3\|x + 2\| - 6$	$f(x) = \frac{1}{2}\|x - 4\| + 3$	$f(x) = -3\|5 - x\|$
Dom f	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
Ran f	$]-\infty, 4]$	$[-6, +\infty[$	$[3, +\infty[$	$]-\infty, 0]$
Zero(s) if they exist	*−1 and 3*	*−4 and 0*	*None*	*5*
Initial value	*2*	*0*	*5*	*−15*
Sign	$f(x) \geq 0$ over $[-1, 3]$ $f(x) < 0$ over $]-\infty, -1[\cup]3, +\infty[$	$f(x) \geq 0$ over $]-\infty, -4] \cup [0, +\infty[$ $f(x) < 0$ over $]-4, 0[$	$f(x) \geq 0$ over $\mathbb{R}$ $f(x) < 0$ never	$f(x) \geq 0$ over $\{5\}$ $f(x) < 0$ over $\mathbb{R} \setminus \{5\}$
Variation	$f \nearrow$ over $]-\infty, 1]$ $f \searrow$ over $[1, +\infty[$	$f \nearrow$ over $[-2, +\infty[$ $f \searrow$ over $]-\infty, -2]$	$f \nearrow$ over $[4, +\infty[$ $f \searrow$ over $]-\infty, 4]$	$f \nearrow$ over $]-\infty, 5]$ $f \searrow$ over $[5, +\infty[$
Extrema	*max = 4*	*min = −6*	*min = 3*	*max = 0*

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20. Find the rule of an absolute value function whose graph

a) has the vertex V(3, 4) and passes through the point P(7, 6). $y = \frac{1}{2}|x - 3| + 4$

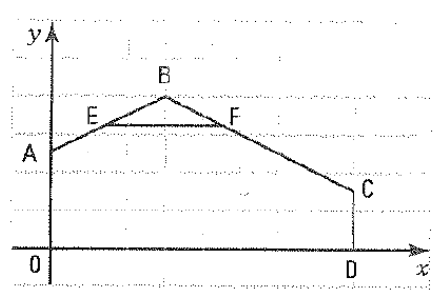
b) passes through the points A(2, −6), B(5, −8) and C(−4, −6). $y = -\frac{2}{3}|x + 1| - 4$

c) passes through the points A(1, −1), B(3, −5) and C(−4, −3). $y = -2|x + 1| + 3$

21. In order to draw the simulated trajectory of a toy airplane, Ethan uses the rule of an absolute value function that gives the airplane's height y, in metres, as a function of elapsed time x, in seconds. The rule of the function is $y = -\frac{5}{4}|x - 8| + 10$.

For how many seconds is the height of the airplane above 7 m? *4.8 seconds*

22. In the Cartesian plane on the right, a view of an airplane hangar is represented with the roof of the hangar corresponding to an absolute value function given by the rule $y = -\frac{1}{2}|x - 6| + 8$.



a) What is the height of the wall A0? *5 m*

b) What is the height of the wall CD if the width of the hangar is equal to 16 m? *3 m*

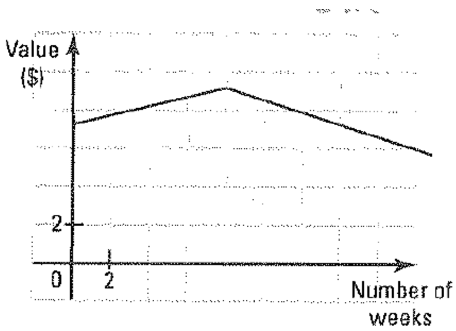
c) The ceiling EF is built at a height of 6.5 m. What is the width of the ceiling? ~~5.6 m~~ ? 6 m

23. The graph on the right represents the evolution of a share's value on the stock market. Eight weeks after its purchase, the share reaches its maximum value of $9. If it initially was worth $7, what will it be worth after 13 weeks?

$y = a|x - 8| + 9$; $7 = 8a + 9$; $a = -\frac{1}{4}$

$y = -\frac{1}{4}|x - 8| + 9$.

It will be worth $7.75.



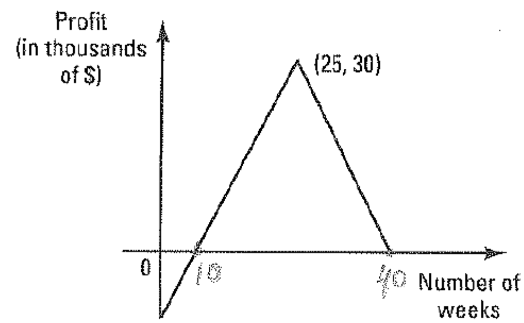
24. The graph on the right represents the profit of a recycling company during its first 40 weeks of operation.

During how many weeks was the profit greater than $15 000?

$y = -2|x - 25| + 30$

$-2|x - 25| + 30 = 15$; $x = 17.5$ or $x = 32.5$.

During 15 weeks.



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25. The air conditioning system in an office building has been programmed so that it turns on when the outside temperature reaches 23 °C and turns off when it reaches 20 °C.

The outside temperature varies according to the rule of the absolute value function given by $y = -3|x - 6| + 35$ where x represents the elapsed number of hours since 6 a.m. and y represents the outside temperature in °C.

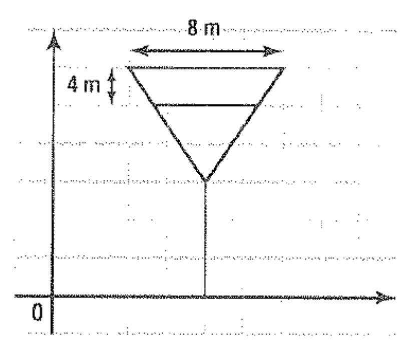
How many hours was the system on?

It turns on at 8 a.m. and turns off at 5 p.m. The system is on during 9 hours.

26. The lateral view of a channelling system is represented in the Cartesian plane on the right, scaled in metres.

The walls of this system are represented by an absolute value function with the rule: $y = 3|x - 8| + 12$.

A filtering net is placed 4 m below the ceiling of the canal. If the width of the canal is 8 m, what is the width of the filtering net?



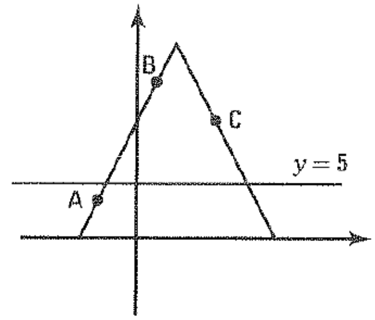
When $x = 12$, $y = 24$;

When $y = 20$, $x = \frac{16}{3}$ or $x = \frac{32}{3}$. The width of the net is 5.33 m.

27. The graph on the right represents the front of a house. The base of the roof corresponds to the line $y = 5$.

The sides of the roof form the graph of an absolute value function passing through the points A(−2, 3), B(2, 13) and C(8, 8).

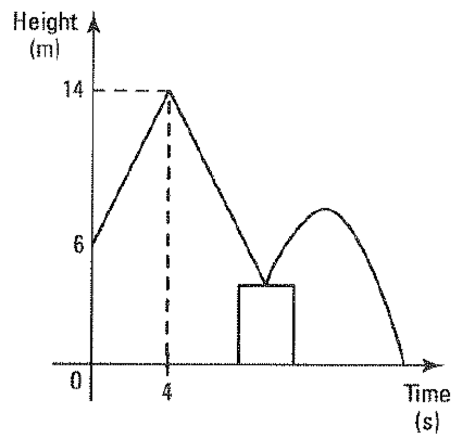
What is the area of the triangle limited by the roof and the line?



$y = -\frac{5}{2}|x - 4| + 18$; base = $\frac{52}{5}$; height = 13.

The area of the triangle is 67.6 u^2.

28. A projectile is thrown from a height of 6 m and follows the trajectory of an absolute value function. It reaches a maximum height of 14 m after 4 seconds. Five seconds after reaching its maximum height, it bounces off a cement block and follows the trajectory of a quadratic function. If the maximum height of the second bounce is 8 m and occurs three seconds after bouncing off the cement block, when will the projectile hit the ground? (Round your answer to the nearest second).



$y = -2|x - 4| + 14$, P(9, 4); $y = -\frac{4}{9}(x - 12)^2 + 8$;

The projectile hits the ground at t = 16 s.