5. Write the rules of the square root functions in the form $y = a\sqrt{x - h} + k$ or $y = a\sqrt{-(x - h)} + k$.

a) $y = -2\sqrt{4x + 8} + 3$

$y = -4\sqrt{x + 2} + 3$

b) $y = 2\sqrt{9x - 36} + 4$

$y = 6\sqrt{x - 4} + 4$

c) $y = -\frac{1}{2}\sqrt{18 - 9x} + 1$

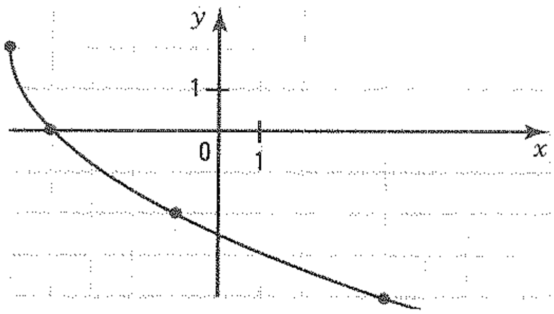
$y = -\frac{3}{2}\sqrt{-(x - 2)} + 1$

d) $y = -\frac{3}{4}\sqrt{2 - 4x} + 7$

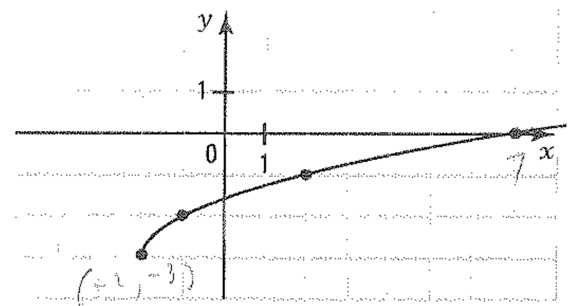
$y = -\frac{3}{2}\sqrt{-\left(x - \frac{1}{2}\right)} + 7$

6. Represent the following square root functions in the Cartesian plane.

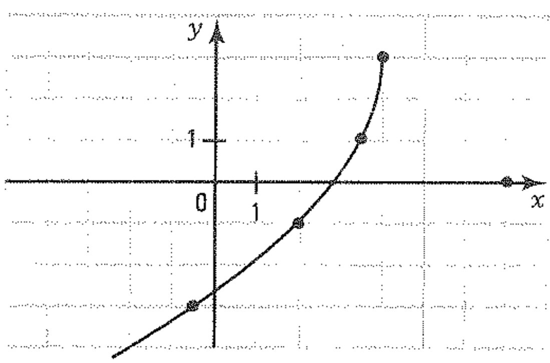
a) $y = -2\sqrt{x + 5} + 2$



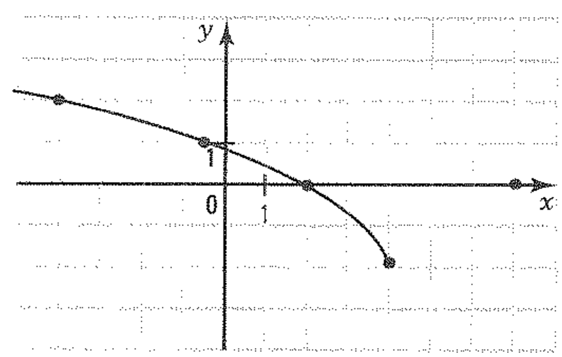
b) $y = \frac{1}{2}\sqrt{4x + 8} - 3$



c) $y = -2\sqrt{-2(x - 4)} + 3$

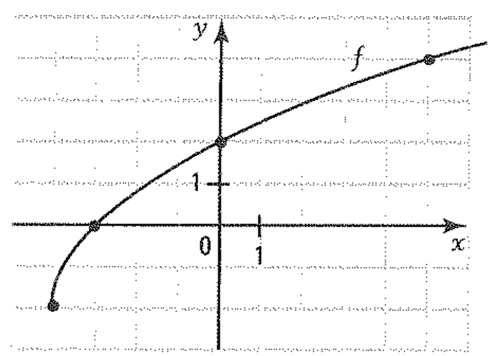


d) $y = \sqrt{-2(x - 4)} - 2$



7. Consider the function $f(x) = 2\sqrt{x + 4} - 2$.

a) Graph the function f.



b) Study the function f.

dom $= [-4, +\infty[$ *ran* $= [-2, +\infty[$

Zero: -3; *initial value:* 2

$f(x) \geqslant 0$ over $[-3, +\infty[$; $f(x) \leqslant 0$ over $[-4, -3]$

$f \nearrow, \forall x \in$ dom f

min $f = -2$

c) Using the graph of f, solve the inequality

1. $f(x) \geqslant 2$ $[0, +\infty[$

2. $f(x) \leqslant 4$ $[-4, 5]$

8. Determine the domain and range of the following functions.

a) $y = -2\sqrt{6 - 3x} + 4$

dom =]−∞, 2]; ran =]−∞, 4]

b) $y = 3\sqrt{4x + 2} - 1$

dom $= \left[-\frac{1}{2}, +\infty\right[$; *ran = [−1, +∞[*

9. Determine the zero and initial value of the following functions.

a) $y = -3\sqrt{6 - 4x} + 9$

zero: $-\frac{3}{4}$, *i.v.:* $-3\sqrt{6} + 9$

b) $y = 2\sqrt{4x - 1} - 1$

zero: $\frac{5}{16}$, *i.v.: does not exist*

c) $y = 2\sqrt{x - 5} + 4$

No zero, i.v.: does not exist

d) $y = -2\sqrt{3x + 1}$

zero: $-\frac{1}{3}$, *i.v.: −2*

10. Consider the absolute value function $f(x) = -2|6 - 2x| + 8$ and the square root function $g(x) = 3\sqrt{\frac{1}{2}(x + 4)} - 5$. Determine

a) $g \circ f(4) =$ *1*

b) $f \circ g(-2) =$ *−12*

11. Determine the interval over which each of these functions is positive.

a) $f(x) = 3\sqrt{x + 5} - 6$

f(x) ≥ 0 over [−1, +∞[

b) $f(x) = -2\sqrt{6 + 4x} + 4$

f(x) ≥ 0 over $\left[-\frac{3}{2}, -\frac{1}{2}\right]$

c) $f(x) = \frac{1}{2}\sqrt{4 - x} + 5$

f(x) ≥ 0 over]−∞, 4]

d) $f(x) = -3\sqrt{-2x + 8} - 1$

f(x) is never positive

12. Solve the following inequalities.

a) $-2\sqrt{x + 3} + 2 \geq 0$

S = [−3, −2]

b) $\sqrt{3x + 4} < -1$

S = ∅

c) $5\sqrt{2 - x} > 4$

S $= \left]-\infty, \frac{34}{25}\right[$

d) $\sqrt{\frac{1}{2}x + 8} > 0$

S =]−16, +∞[

13. Determine the interval over which each of these functions is increasing.

a) $f(x) = 3\sqrt{-2(x - 1)} + 5$

f is never increasing

b) $f(x) = -2\sqrt{-3(x + 4)}$

f ↗ over]−∞, −4]

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14. Study each of the following functions and complete the following table.

	$f_1(x) = 3\sqrt{x-2} - 1$	$f_2(x) = -2\sqrt{\frac{1}{2}(x+4)} + 6$	$f_3(x) = \sqrt{2-x} + 1$	$f_4(x) = -2\sqrt{-x} + 4$
Domain	$[2, +\infty[$	$[-4, +\infty[$	$]-\infty, 2]$	$]-\infty, 0]$
Range	$[-1, +\infty[$	$]-\infty, 6]$	$[1, +\infty[$	$]-\infty, 4]$
Zero	$\frac{19}{9}$	14	does not exist	−4
Initial value	does not exist	$-2\sqrt{2} + 6$	$\sqrt{2} + 1$	4
Sign	$f(x) \geq 0$ over $\left[\frac{19}{9}, +\infty\right[$ $f(x) < 0$ over $\left[2, \frac{19}{9}\right[$	$f(x) \geq 0$ over $[-4, 14]$ $f(x) < 0$ over $]14, +\infty[$	$f(x) \geq 0$ over $]-\infty, 2]$ $f(x) < 0$ never	$f(x) \geq 0$ over $[-4, 0]$ $f(x) < 0$ over $]-\infty, -4[$
Variation	$f \nearrow$ over $[2, +\infty[$ $f \searrow$ never	$f \nearrow$ never $f \searrow$ over $[-4, +\infty[$	$f \nearrow$ never $f \searrow$ over $]-\infty, 2]$	$f \nearrow$ over $]-\infty, 0]$ $f \searrow$ never
Extrema	min = −1	max = 6	min = 1	max = 4

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15. Find the rule of each of the square root functions given its vertex V and a point P on its graph.

a) V(5, 3) and P(9, 3.5)

$y = \frac{1}{4}\sqrt{x-5} + 3$

b) V(−2, −1) and P(−6, −4)

$y = -\frac{3}{2}\sqrt{-(x+2)} - 1$

c) V(−2, 4) and P(23, 2)

$y = -\frac{2}{5}\sqrt{x+2} + 4$

d) V(5, 3) and P(−13, 5)

$y = \frac{1}{3}\sqrt{-2(x-5)} + 3$

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16. Determine the rule of the inverse of the following functions and indicate the domain of the inverse.

a) $y = 2\sqrt{x-1} + 7$

$y = \frac{1}{4}(x-7)^2 + 1$; dom = $[7, +\infty[$

b) $y = -3\sqrt{x+4} - 1$

$y = \frac{1}{9}(x+1)^2 - 4$; dom = $]-\infty, -1]$

c) $y = 4\sqrt{-(x+3)} - 2$

$y = -\frac{1}{16}(x+2)^2 - 3$; dom = $[-2, +\infty[$

d) $y = -2\sqrt{-(x-5)} + 4$

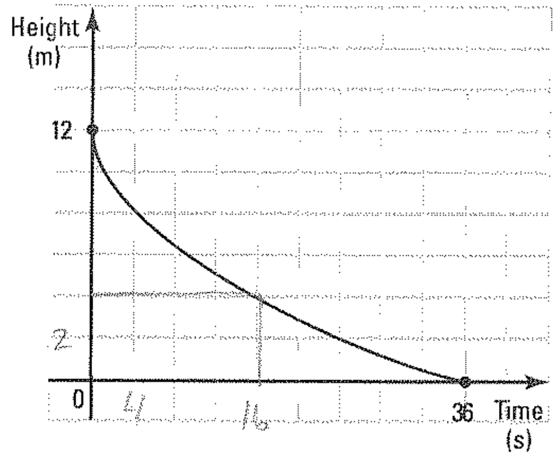
$y = -\frac{1}{4}(x-4)^2 + 5$; dom = $]-\infty, 4]$

17. At a water park, Raphael is getting ready to go down a slide.

The function f represented on the right gives Raphael's height h (in m) as a function of elapsed time t (in s) since his departure.

At what instant will he be at a height of 4 m?

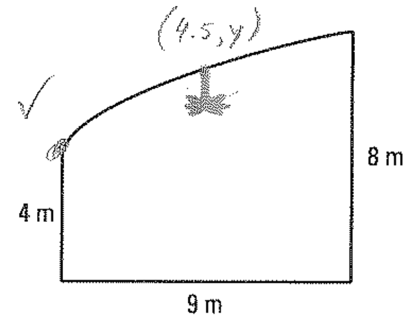
$h(t) = -2\sqrt{t} + 12$; after 16 seconds.



18. The lateral view of a solarium is represented by the graph on the right where the glass ceiling follows the curve of a square root function. A light is located at the centre of the room as indicated in the figure. Determine at what height the base of the light is located. (Round your answer to the nearest tenth.)

V(0, 4); $y = a\sqrt{x} + 4$; P(9, 8); $y = \frac{4}{3}\sqrt{x} + 4$.

When x = 4.5, the height is y = 6.8 m.



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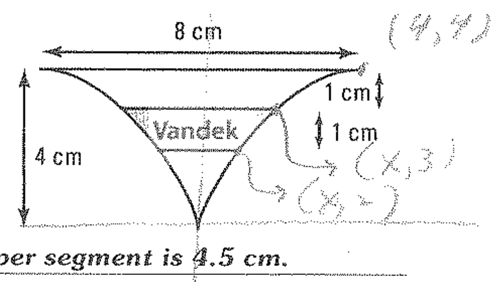
19. A company's logo is drawn using the graphs of two square root functions as illustrated in the figure on the right. The company's name is limited by two line segments.



a) What is the length of the upper segment?

Rule: $y = 2\sqrt{x}$. When y = 3, x = 2.25 cm. The length of the upper segment is 4.5 cm.

b) What is the length of the lower segment? ***2 cm***

$y = a\sqrt{x - h} + k$

$4 = a\sqrt{4 - 0} + 0$

$4 = 2a$

$a = 2$

a) $y = 2\sqrt{x}$

$3 = 2\sqrt{x}$

$9 = 4x$

$\frac{9}{4} = x$

$2 \times \frac{9}{4} = \frac{9}{2} =$ 4.5 cm

b) $2 = 2\sqrt{x}$

$1 = \sqrt{x}$

$x = 1$

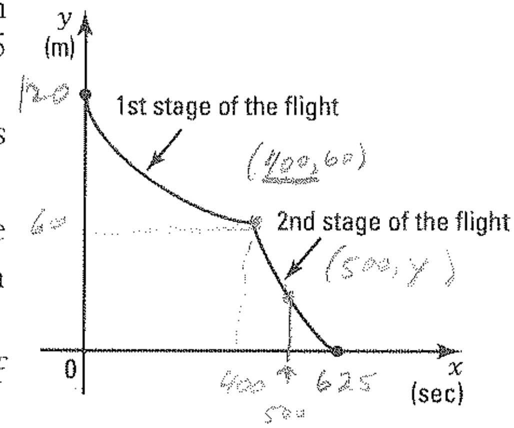
2 cm

20. The flight of a bird is observed from its takeoff at $t = 0$ from a 150 m high tower until it reaches the ground at $t = 625$ seconds.

The bird's flight is described by two square root functions represented in the figure on the right.

The 1st stage of its flight lasts 400 s and is described by the rule $y = -3\sqrt{t} + 120$ where t represents the time, in seconds, and y the height of the bird, in metres.

At the instant $t = 400$ s, the bird begins the second stage of its flight.



At what height will the bird be 500 s after the beginning of its flight?

$y = -4\sqrt{t - 400} + 60$. It will be at a height of 20 m.

$y = -3\sqrt{t} + 120$ (150?)

$y = -3\sqrt{400} + 120$

$y = 60$

$y = a\sqrt{x - h} + k$

$0 = a\sqrt{625 - 400} + 60$

$-60 = a(15)$

$-4 = a$

$y = -4\sqrt{x - 400} + 60$

$y = -4\sqrt{500 - 400} + 60$

$= -4\sqrt{100} + 60$

$= 20$ m