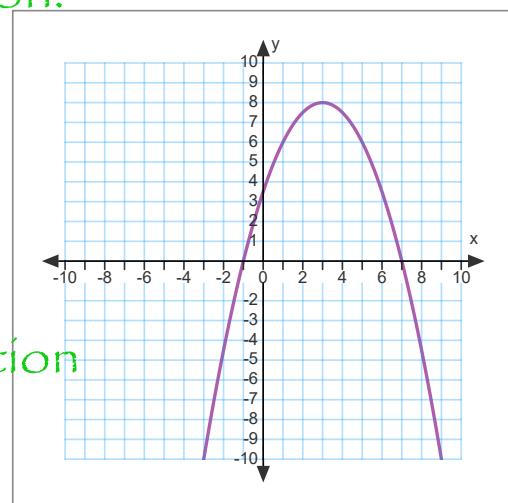


Square Root Function

Recall: The inverse of a second-degree function,
eg., $f(x) = -\frac{1}{2}(x-3)^2 + 8$ is a sideways
parabola ~ not a function.

However, if we consider only
one-half of the roots, it is a
function - a Square Root Function



Square Root Function

Basic Function

Rule: $f(x) = \sqrt{x}$

x							
y							

Graph:

Properties:

Domain:

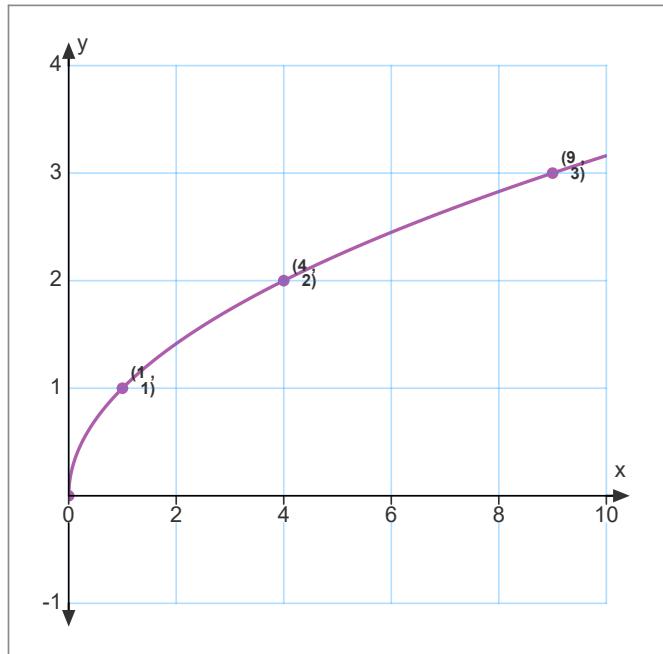
Range:

Variation:

Sign:

Extrema:

Intercepts:

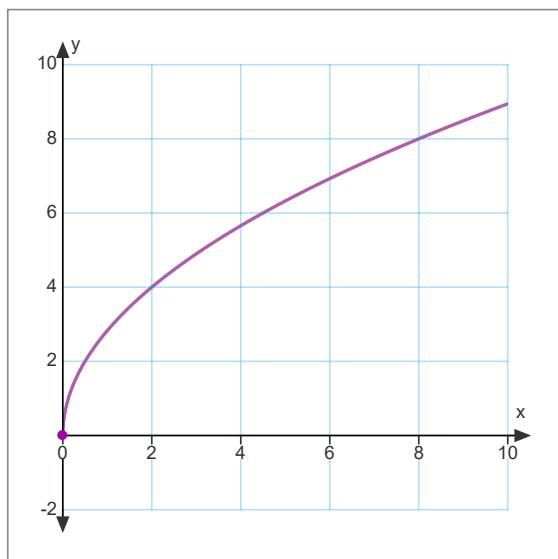


Parameters a & b

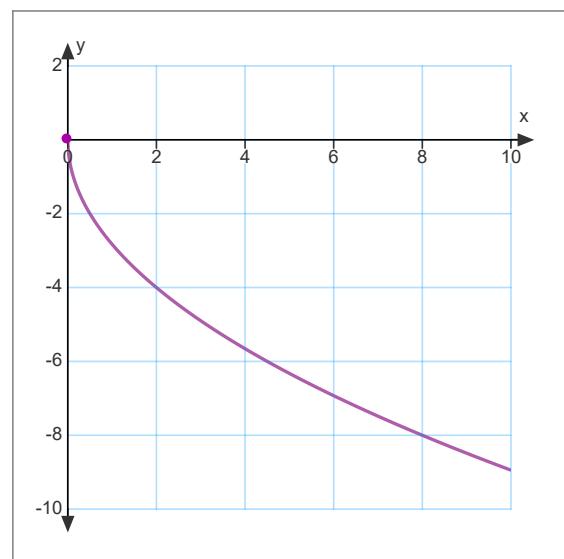
Rule: $f(x) = a\sqrt{bx}$

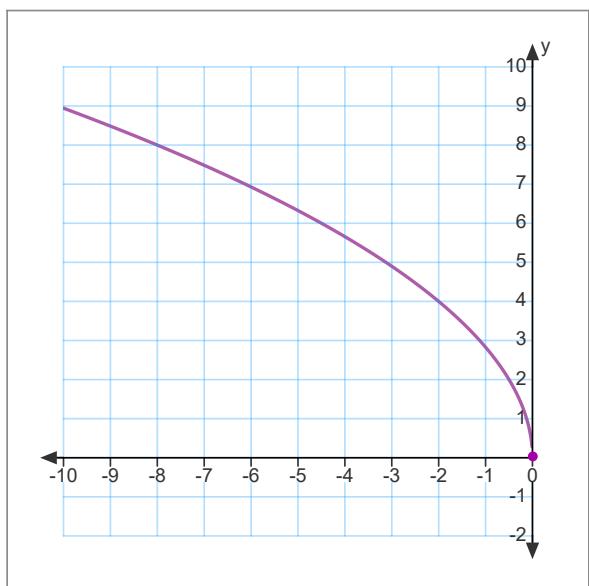
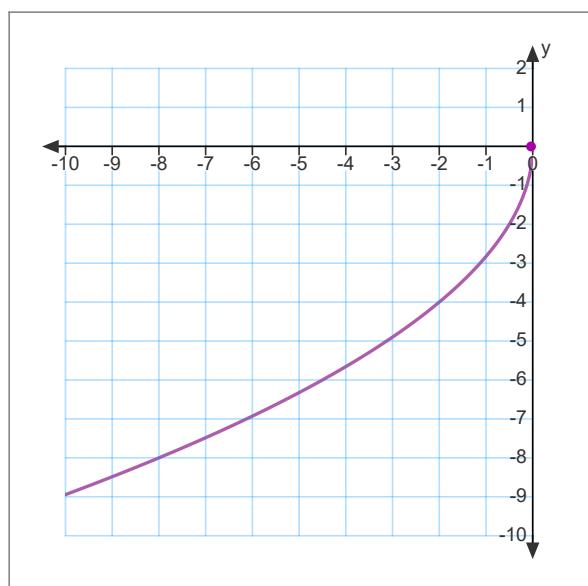
There are four cases:

a^+, b^+



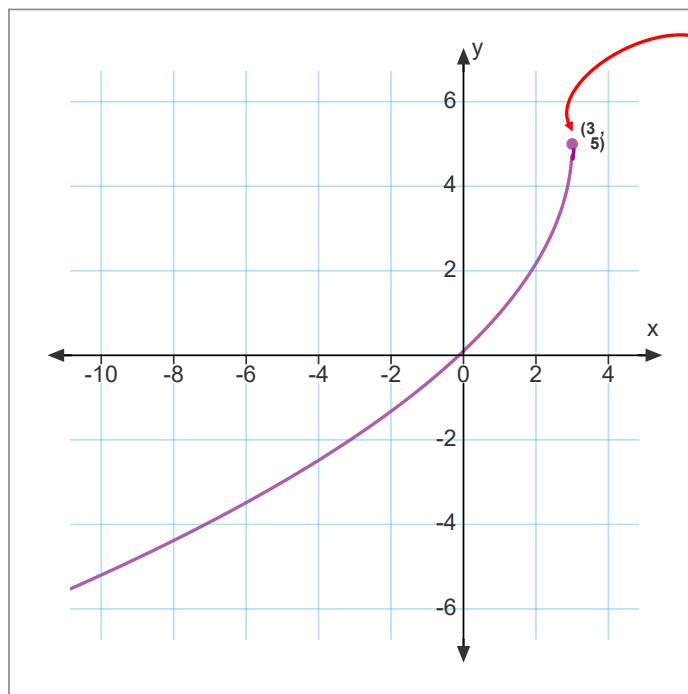
a^-, b^+



a^+, b^-  a^-, b^- 

Transformed Function

$$f(x) = a\sqrt{b(x-h)} + k$$



vertex or
starting/ending
point moves to
(h, k)

Parameter b can be factored out and a "new a " can be created. However, b does not entirely disappear. b will transform in to ± 1 .

Examples:

$$f(x) = -3\sqrt{4x+8} - 1$$

$$f(x) = \frac{1}{3}\sqrt{27-9x} + 5$$

Write in standard form.

$$1. \ f(x) = 3\sqrt{-4x+8} - 3$$

$$2. \ g(x) = 5\sqrt{48-16x} + 1$$

$$3. \ h(x) = 11\sqrt{6-2x} - 5$$

The value of b affects the domain of the function.

Examples: $f(x) = 3\sqrt{-5(x-2)} - 1$

Dom:

$$f(x) = -\frac{3}{4}\sqrt{2(x+4)} + 6$$

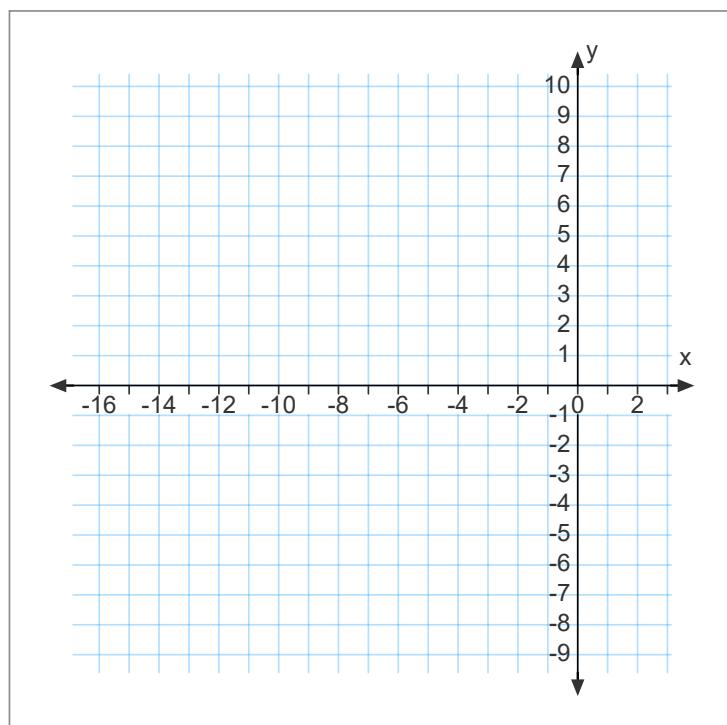
Dom:

Graphing a Square Root Function

- make a table

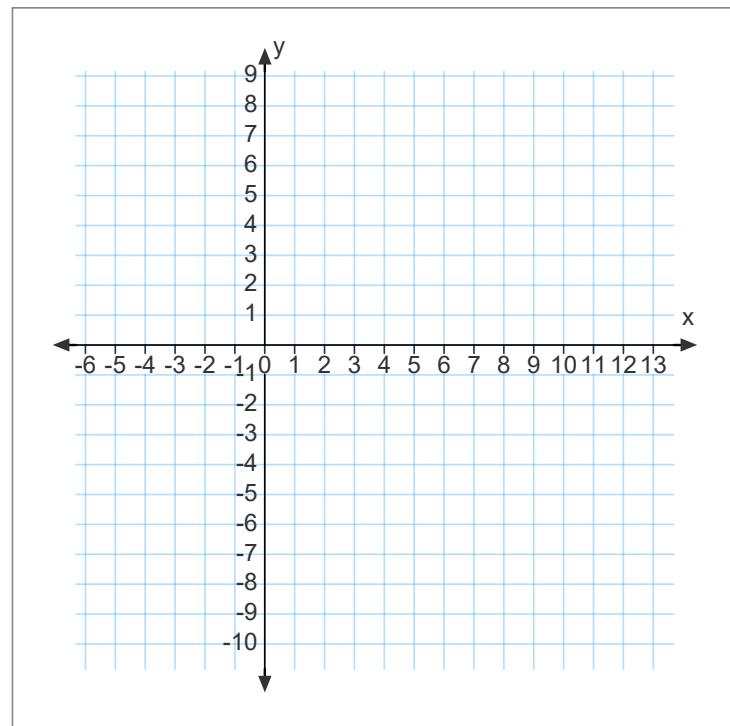
Example: Graph the function $f(x) = -2\sqrt{-(x+4)} + 3$

x	y



Example: Graph the function $f(x) = \frac{1}{2}\sqrt{4x-12} - 1$

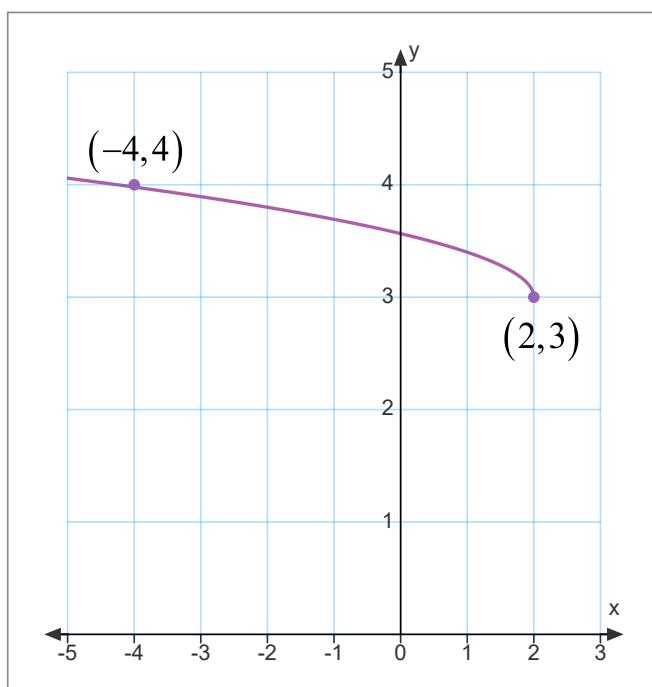
x	y



Finding the Rule

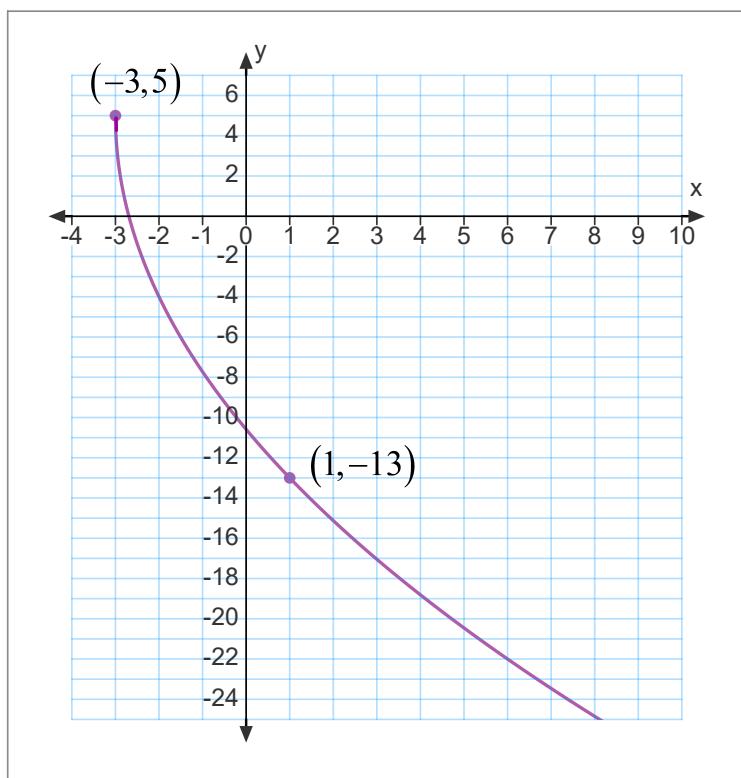
- Given the vertex and a point.

Example: Determine the rule.

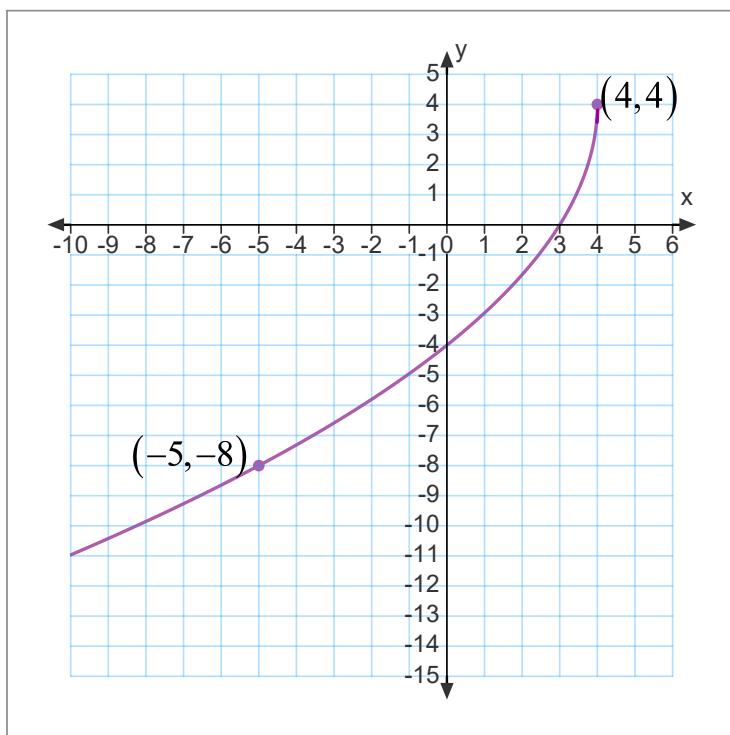


$$\text{Form: } f(x) = a\sqrt{\pm 1(x-h)} + k$$

Example: Determine the rule.



Example: Determine the rule.



If $f(x) = 3\sqrt{12 - 4x} - 5$

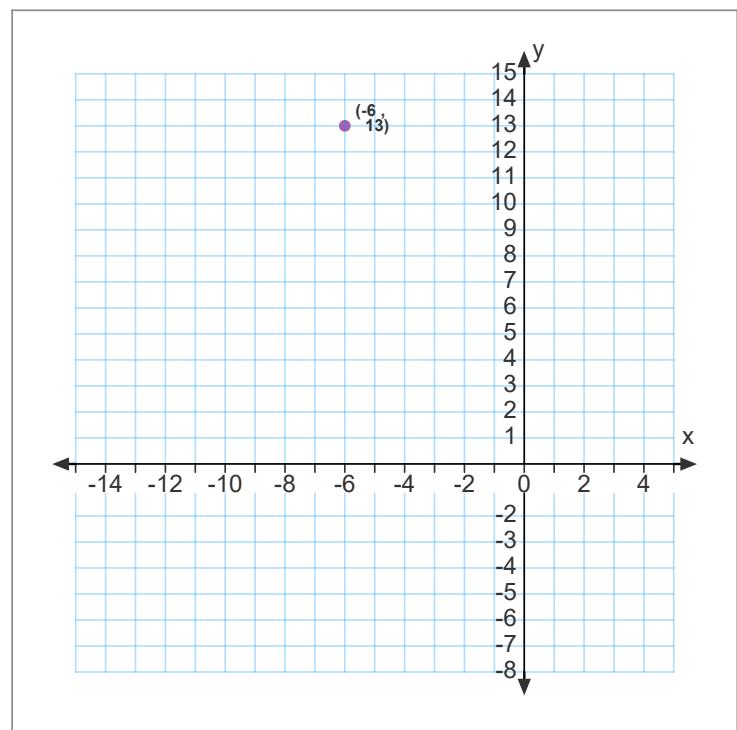
- a) What are the domain and range of f ?
- b) Draw the graph of f .

a)

Dom:

Ran:

b)



Zeros, Equations and Inequalities

Example: The hull of a ship follows the curve of the function $f(x) = -2\sqrt{x-3} + 4$ for a length of 18m. The water line corresponds to the x -axis.

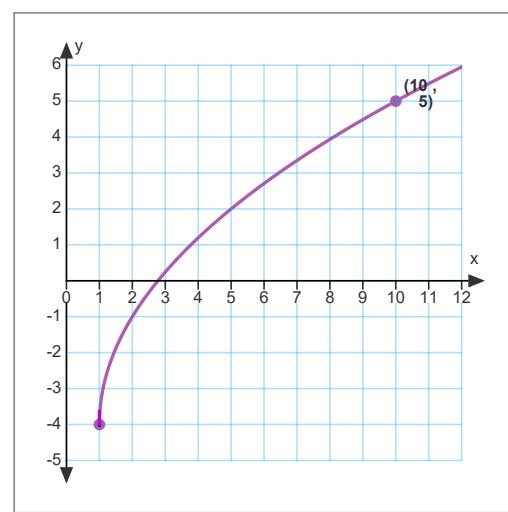
- a) What interval represents the part of the hull that is above the water line?

a) $f(x) = -2\sqrt{x-3} + 4$

- b) When carrying a full cargo, the equation becomes $g(x) = -2\sqrt{x-3} + 2$. What interval represents the portion of the hull below the water line?

Example: After one month in operation, a company's revenue has grown according to the square root function shown.

Losses amounted to \$4 million the first month. After ten months, a \$5 million profit was recorded.



After how many months in operation did the company begin to make a profit?

Determine the zero of the function $f(x) = -3\sqrt{x+7} - 6$.

No solution - only a positive square root is allowed.

Provide a study of the following function.

$$f(x) = -3\sqrt{-4(x-7)} + 6$$

Dom:

Ran:

Inc:

Dec:

Pos:

Neg:

Zero:

y-int:

Solve for x .

1. $2\sqrt{2x+4} - 6 = 0$

$$2. \sqrt{2x - 6} = 4$$

$$3. \sqrt{x + 6} = \sqrt{x^2 - 4x}$$

$$4. -3\sqrt{-2(x-4)} + 9 \leq -6$$

$$5. \quad 10 > 4\sqrt{4x-8} - 6$$

$$6. \quad -2\sqrt{4x-1} + 7 \leq 3$$

Inverse of the Square Root Function

Example: What is the rule of the inverse of the function $f(x) = 2\sqrt{-(x-4)} + 6$

Dom f :

Dom f^{-1} :

Ran f :

Ran f^{-1} :

$$\begin{aligned}
 f(x) &= 2\sqrt{-(x-4)} + 6 \\
 x &= 2\sqrt{-(y-4)} + 6 \quad \text{isolate } y \\
 x - 6 &= 2\sqrt{-(y-4)}
 \end{aligned}$$

$$\begin{array}{lll}
 \frac{1}{2}(x-6) &= \sqrt{-(y-4)} & \text{or} & 0.5x - 3 = \sqrt{-(y-4)} \\
 \frac{1}{4}(x-6)^2 &= -(y-4) & (0.5x-3)^2 &= -(y-4) \\
 -\frac{1}{4}(x-6)^2 &= y-4 & 0.25x^2 - 3x + 9 &= -(y-4) \\
 -\frac{1}{4}(x-6)^2 + 4 &= y & -0.25x^2 + 3x - 9 &= y-4 \\
 && -\frac{1}{4}x^2 + 3x - 5 &= y
 \end{array}$$

Therefore, the inverse of $f(x) = 2\sqrt{-(x-4)} + 6$ is

$$f^{-1}(x) = -\frac{1}{4}(x-6)^2 + 4, \text{ where } x \in [6, +\infty[$$

or

$$f^{-1}(x) = -\frac{1}{4}x^2 + 3x - 5, \text{ where } x \geq 6$$

There is another way.... $f(x) = 2\sqrt{-(x-4)} + 6$

1. We know that the inverse will be half of a second-degree function.

$$f^{-1}(x) = a(x-h)^2 + k$$

$$f(x): h = \quad f^{-1}(x): h = \\ k = \quad k =$$

Dom $f^{-1}(x)$:

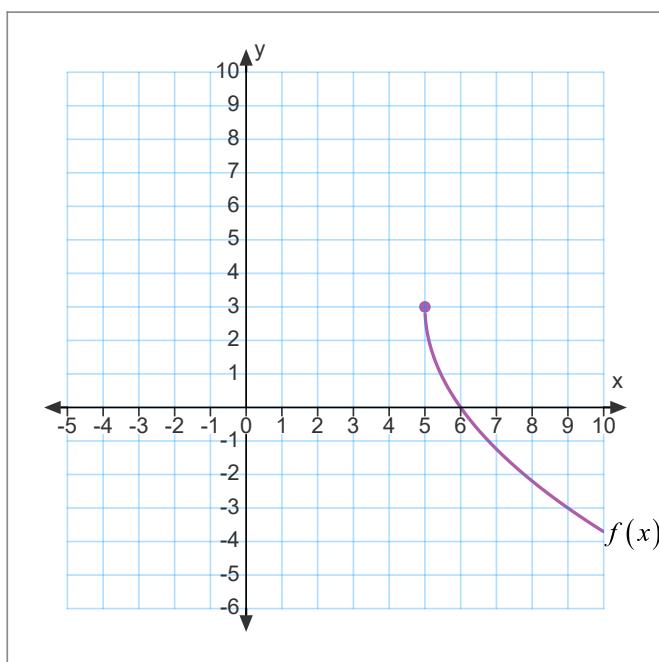
2. Determine a point on $f(x)$; switch the coordinates to get a point on $f^{-1}(x)$. Use that point to determine the value of a .

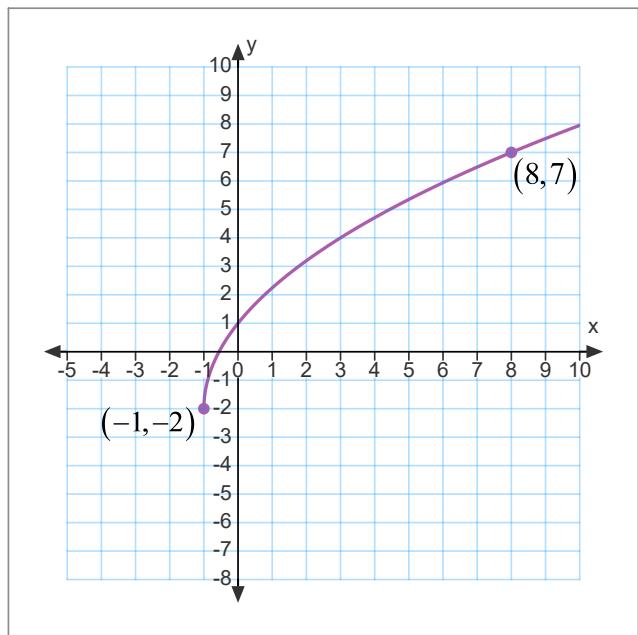
$$f(x) = 2\sqrt{-(x-4)} + 6$$

Let $x =$

Examples:

Given $f(x) = -3\sqrt{x-5} + 3$, sketch the inverse and find its rule.





Sketch the inverse

Determine...

- the domain and range of the inverse.
- the rule of the function.
- the rule of the inverse.