Semi-Linear Systems

aka: a system of first-degree (line) and second-degree (parabola) equations.

Example: $y = x^2 - 6x + 11$

$y = 2x - 4$

We have to find the point(s) of intersection between the line and parabola.

Algebraically: (by comparison)

$$y = x^2 - 6x + 11$$
$$y = 2x - 4$$

$x^2 - 6x + 11 = 2x - 4$
$-2x \quad -2x$
$+4 \quad +4$

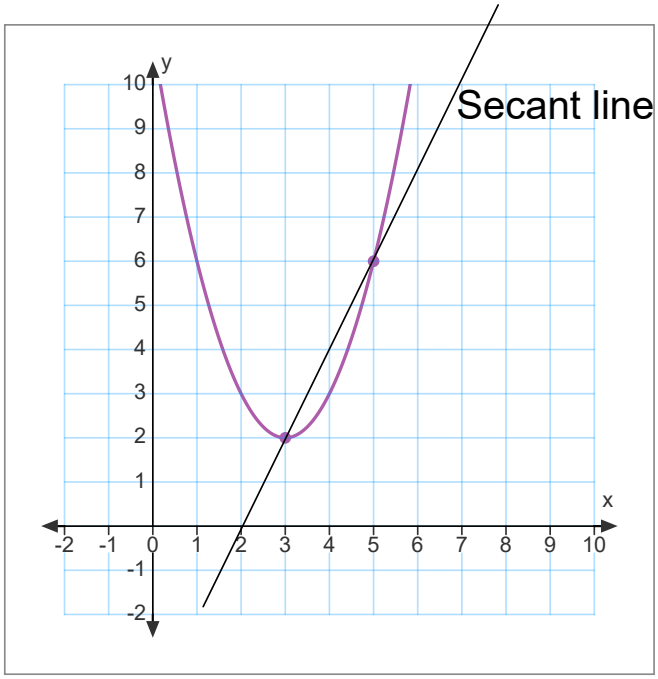
① factor
② quadratic formula

$x^2 - 8x + 15 = 0$
$(x-5)(x-3) = 0$

$x-5=0$ or $x-3=0$
$x_1 = 5$ $x_2 = 3$
$y_1 = 2(5)-4$, $y_2 = 2(3)-4$
$= 6$ $= 2$ ✓

$(5,6)$ & $(3,2)$

Graphically:



There are two solutions

Example: $y = -x^2 + 2x + 3$
$y = -2x + 7$

by comparison

$-x^2 + 2x + 3 = -2x + 7$
$+2x \quad +2x$
$-7 \quad -7$

$y = -(4) + 4 + 3$
$= 3$

$y = -2(2) + 7$
$= -4 + 7$
$= 3$

$(-x^2 + 4x - 4 = 0) \cdot -1$

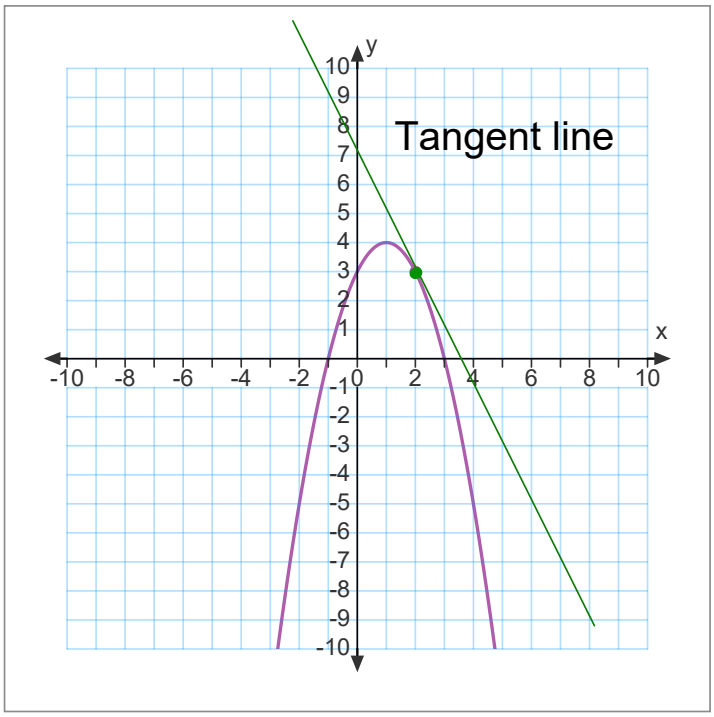
P.S.T $\leftarrow x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

$(2, 3)$

Graphically:



There is one solution (the line is tangent to the parabola).

Example: $y = x^2 - 6x + 4$

$y = -3x - 1$

$x^2 - 6x + 4 = -3x - 1$

$1x^2 - 3x + 5 = 0$

a b c

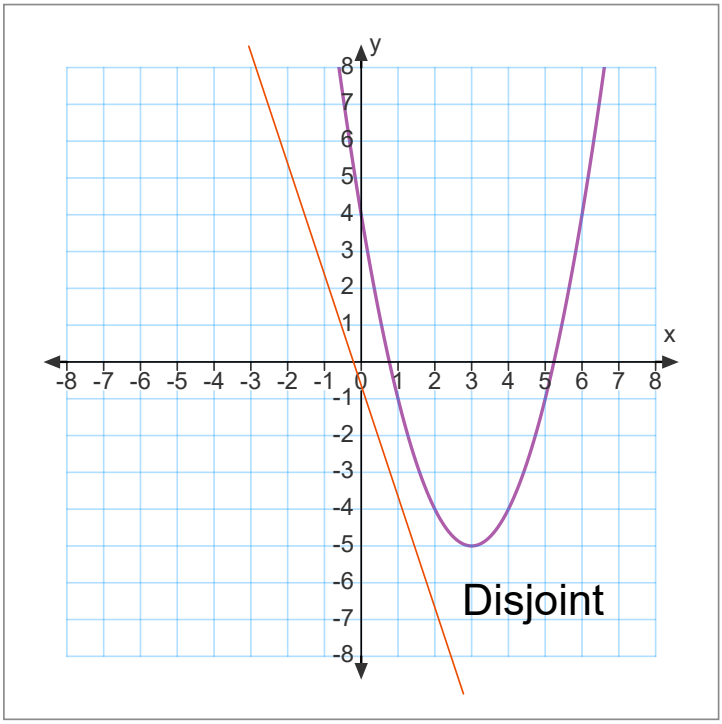
$x = \frac{3 \pm \sqrt{9 - 4(1)(5)}}{2}$

$= \frac{3 \pm \sqrt{-11}}{2}$ not possible

$x = \emptyset$

No solution

Graphically:

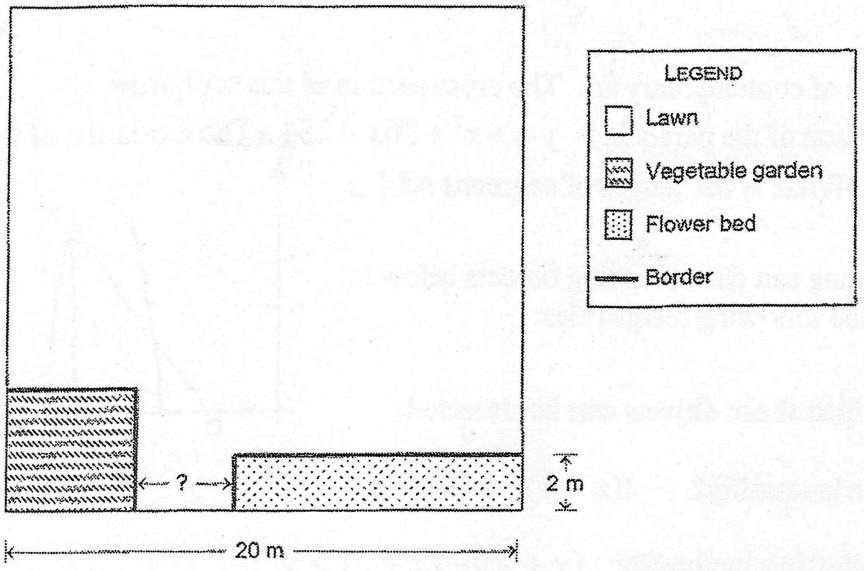


There is no solution.

Summary: There are 3 possible cases when solving a system of a linear equation & second-degree equations...

a) two solutions
b) one solution
c) no solution

Sabrina wants to plant a vegetable garden and a flower bed in the two corners of her back yard.
She sets up some concrete borders on two sides of the vegetable garden and on two sides of the flower bed to separate them from the lawn. The following figure shows the layout of the yard.



- The yard is square.
- The vegetable garden is square.
- the flower bed is rectangular.
- The total length of the concrete borders is 23m.
- The total area of the vegetable garden and the flower bed is 47m^2.

What is the distance between the vegetable garden and the flower bed?

- The total length of the concrete borders is 23*m*.
- The total area of the vegetable garden and the flower bed is 47m^2

x: side length of square y: length of rectangle

$2x + y + 2 = 23$

OR $2x + y = 21$

$x^2 + 2y = 47$

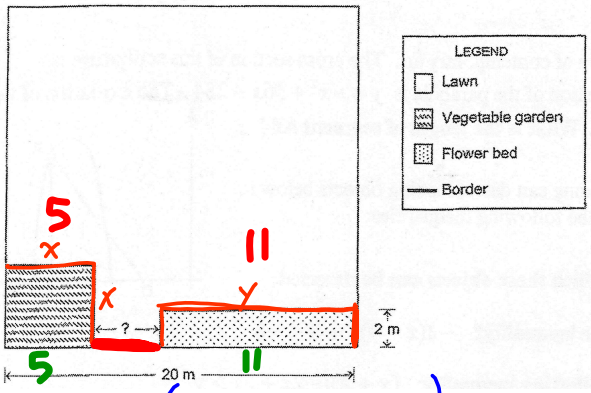
(sub) $y = -2x + 21$

$x^2 + 2y = 47$

$x^2 + 2(-2x + 21) = 47$

$x^2 - 4x + 42 = 47$

$x^2 - 4x - 5 = 0$



Elim $-2 \cdot (2x + y = 21)$

$-4x - 2y = -42$

$+ x^2 + 2y = 47$

$x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$x+1=0$ or $x-5=0$

$x=-1$ (Reject as a length)

$x=5$

$y = -2x + 21$

$y = -10 + 21$

$y = 11$

$x=5$

$y=11$

Answer: $20-(11+5)$

$20-16 = 4\text{m}$