Semi-Linear Systems

aka: a system of first-degree (line) and second-degree (parabola) equations.

Example: $y = x^2 - 6x + 11$

$y = 2x - 4$

We have to find the point(s) of intersection between the line and parabola.

Algebraically: (by comparison)

$$y = x^2 - 6x + 11$$
$$y = 2x - 4$$

$x^2 - 6x + 11 = 2x - 4$
$-2x \quad -2x$
$x^2 - 8x + 11 = -4$
$+4 \quad +4$
$x^2 - 8x + 15 = 0$

① factor?
② Quadratic formula

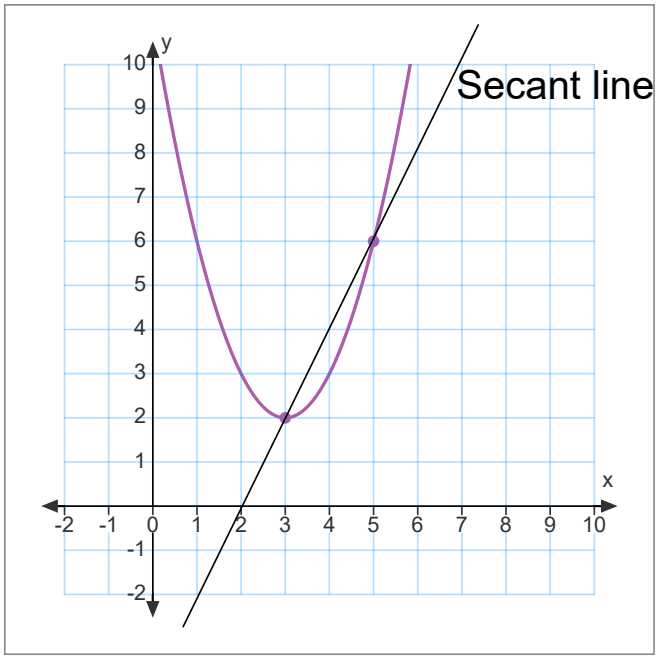
$x^2 - 8x + 15 = 0$
$(x-3)(x-5) = 0$
$x-3=0 \quad x-5=0$
$x=3 \quad x=5$

find both y's

$x_1 = 3$
$y_1 = 3^2 - 6(3) + 11$
$= 9 - 18 + 11$
$= 2$
$(3, 2)$

$x_2 = 5$
$y_2 = 5^2 - 6(5) + 11$
$y_2 = 25 - 30 + 11$
$y_2 = 6$
$(5, 6)$

Graphically:



There are two solutions

Example: $y = -x^2 + 2x + 3$

$y = -2x + 7$

$-2x + 7 = -x^2 + 2x + 3$

$+2x \qquad +2x \qquad -7 \qquad -7$

$0 = -x^2 + 4x - 4$

$0 = x^2 - 4x + 4$

$0 = (x-2)(x-2)$

$x - 2 = 0$

$x = 2$

$y = -2(2) + 7 = 3$

(2, 3)

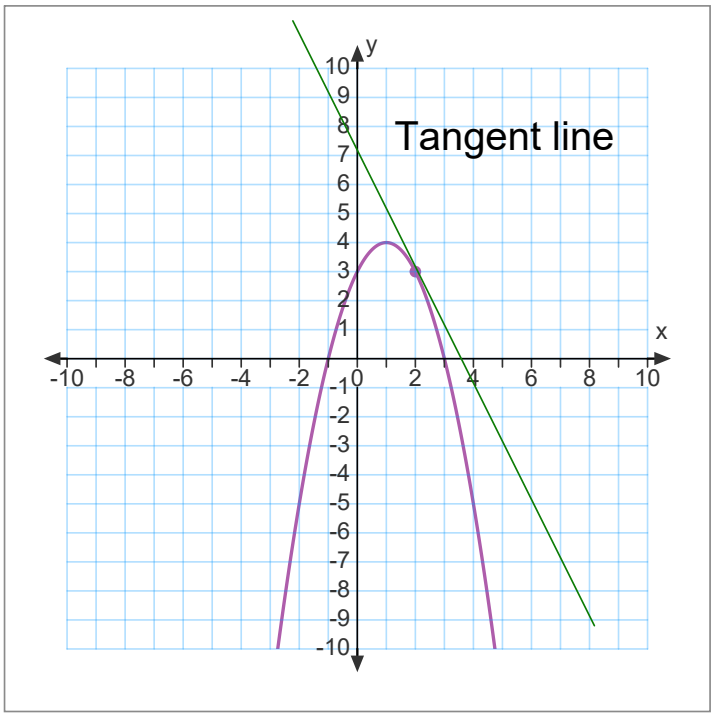
check

$3 = -(4) + 4 + 3$

$3 = 0 + 3$

$3 = 3$ ✓

Graphically:



There is one solution (the line is tangent to the parabola).

Example: $y = x^2 - 6x + 4$

$y = -3x - 1$

$+3x \quad +3x$

$-3x - 1 = x^2 - 6x + 4$

$+1 \quad +1$

$0 = 1x^2 - 3x + 5$

$a \quad b \quad c$

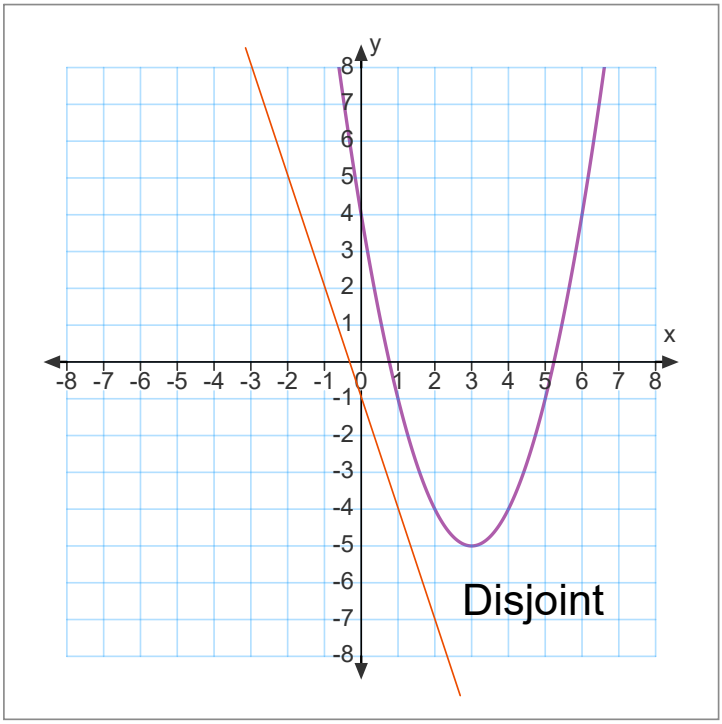
$x = \frac{3 \pm \sqrt{9 - 4(1)(5)}}{2}$

$x = \frac{3 \pm \sqrt{9 - 20}}{2}$

$= \frac{3 \pm \sqrt{-11}}{2}$ can't be done

no solution

Graphically:



There is no solution.

Summary: There are 3 possible cases when solving a system of a linear equation & second-degree equations...

a) two solutions
b) one solution
c) no solution

Word Problems

A father is three times older than his son. The sum of the squares of their ages is 2250. How old are they?

x: father's age

y: son's age

$x = 3y$ (line)

sub

$x^2 + y^2 = 2250$ (not a parabola)

$(3y)^2 + y^2 = 2250$

$9y^2 + y^2 = 2250$

$\frac{10y^2}{10} = \frac{2250}{10}$

$y^2 = 225$

$y = \pm 15$

$y = 15$

$x = 3(15) = 45$

40 + 5

1600 + 400 + 25

check

$(15)^2 + (45)^2 = 2250$

$225 + 2025 = 2250$

$2250 = 2250$

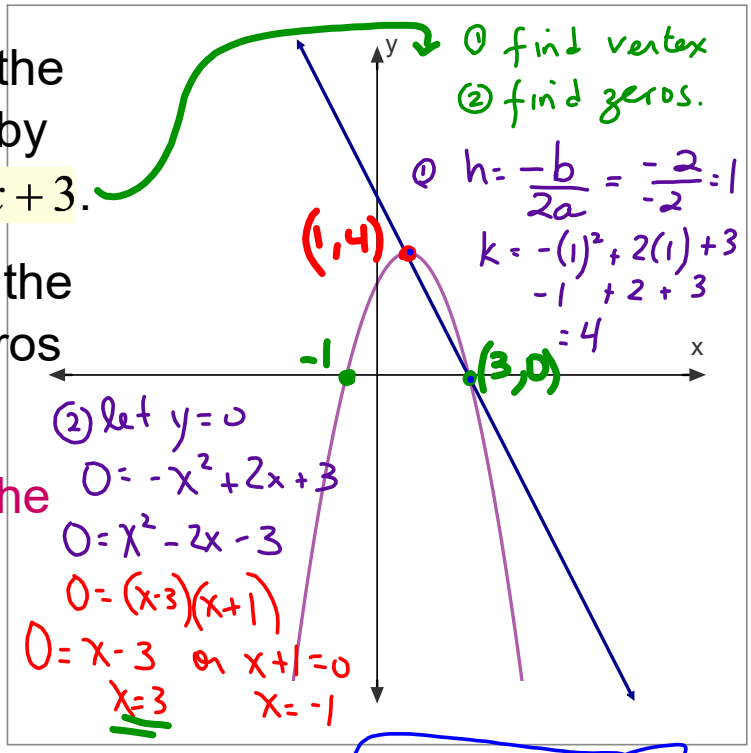
Dad is 45 yrs old

Son is 15.

On the diagram at right, the parabola is represented by the equation $y = -x^2 + 2x + 3$.

The line passes through the vertex and one of the zeros of the parabola.

What is the equation of the line?



① find vertex
② find zeros.

① $h = \frac{-b}{2a} = \frac{-2}{-2} = 1$

$k = -(1)^2 + 2(1) + 3$
$-1 + 2 + 3$
$= 4$

② let $y = 0$

$0 = -x^2 + 2x + 3$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$0 = x - 3$ or $x + 1 = 0$

$x = 3$ $x = -1$

③ equation of line

$m = \frac{0-4}{3-1} = \frac{-4}{2} = -2$

$y = -2x + b$
$4 = -2(1) + b$
$4 = -2 + b$
$6 = b$

④ Answer $y = -2x + 6$