

## Algebraic Method 2: Substitution

Best used when one of the variables is isolated in one equation but not the other. Usually one equation is in standard form and the other is in general form.

Example:  $4x - 2y + 10 = 0$   
 $y = 4x$

Example:  $3x - 5y = 30$   
 $x = 2y - 1$

We substitute what  $y$  is equal to in the other equation, then solve the equation.

$$4x - 2(4x) + 10 = 0$$

$$4x - 8x + 10 = 0$$

$$-4x + 10 = 0$$

$$-4x = -10$$

$$x = 2.5$$

Replace the variable in one of the original equations to find the value of the other variable.

$$y = 4(2.5)$$

$$y = 10$$

check

$$4x - 2y + 10 = 0$$

$$4(2.5) - 2(10) + 10 = 0$$

$$10 - 20 + 10 = 0$$

$$0 = 0$$

**Solution:** (2.5, 10)

example:  $x = -y - 4$   
 $3x + 2y = 5$

$$3(-y - 4) + 2y = 5$$

$$\underline{-3y} - 12 + \underline{2y} = 5$$

$$-y - 12 = 5$$

$$+12 \quad +12$$

$$-y = 17$$

$$\div -1 \quad \div -1$$

$$y = -17$$

$$x = -(-17) - 4$$

$$x = 17 - 4$$

$$x = 13$$

check

$$3(13) + 2(-17) = 5$$

$$39 - 34 = 5$$

$$5 = 5$$

Solution: (13, -17)

example: Solve the system

$$\begin{aligned} 2x + y &= 5 \\ 3x - 2y &= 18 \end{aligned}$$

Neither variable is isolated. But in the first equation, we can see that it would be easy to isolate  $y$ .

So...

$$y = -2x + 5$$

$$3x - 2y = 18$$

Now we can solve using substitution.

$$\frac{7x}{7} = \frac{28}{7}$$

$$x = 4$$

$$3x - 2(-2x + 5) = 18$$

$$3x + 4x - 10 = 18$$

$$7x - 10 = 18$$

$$7x = 28$$

$$y = -2x + 5$$

$$3x - 2y = 18$$

$$3x - 2(-2x + 5) = 18$$

$$3x + 4x - 10 = 18$$

$$7x - 10 = 18$$

$$7x - 10 + 10 = 18 + 10$$

$$7x = 28$$

$$x = 4$$

~~$$y = -2(4) + 5$$~~  
~~$$y = -8 + 5$$~~  
~~$$y = -3$$~~

check  
$$3(4) - 2(-3) = 18$$
  
$$12 + 6 = 18$$
  
$$18 = 18 \checkmark$$

Solution is  $(4, -3)$

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3. a)  $2x - 3y = -7$   
 $y = 2x - 3$

$$2x - 3(2x - 3) = -7$$

$$2x - 6x + 9 = -7$$

$$-4x + 9 = -7$$

$$-4x = -16$$

$$x = 4$$

$$y = 2(4) - 3 = 5$$

$$2(4) - 3(5) = -7$$

$$8 - 15 = -7$$

$$-7 = -7$$

$(4, 5)$

b)  $y = 2x - 5$

$$2x - 5y = 9$$

$$2x - 5(2x - 5) = 9$$

$$2x - 10x + 25 = 9$$

$$-8x + 25 = 9$$

$$-8x = -16$$

$$x = 2$$

$$y = 2(2) - 5$$

$$= -1$$

check

$$2(2) - 5(-1) = 9$$

$$4 + 5 = 9$$

$$9 = 9$$

$(2, -1)$

c)  $x = 3y + 1$

$$2x - 5y = 3$$

$$2(3y + 1) - 5y = 3$$

$$6y + 2 - 5y = 3$$

$$y + 2 = 3$$

$$y = 1$$

$$x = 3(1) + 1$$

$$= 4$$

$$2(4) - 5(1) = 3$$

$$8 - 5 = 3$$

$$3 = 3$$

$(4, 1)$

Example: A father is 9 years younger than 4 times his daughter's age. The difference of their ages is 27.

How old is each person?

① Variables: Father's age:  $y$

Daughter's age:  $x$

② System of Equations:  $y = 4x - 9$   
 $y - x = 27$

③ Solve the system

④ Answer the question

Daughter is 12 yrs old  
 & father is 39.

$$4x - 9 - x = 27$$

$$3x - 9 = 27$$

$$3x = 36$$

$$x = 12$$

$$y = 4(12) - 9 = \underline{\underline{39}}$$

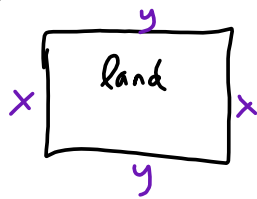
$$39 - 12 = 27$$

$$27 = 27$$

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5.

a)



The length is 10m more than twice the width.

The perimeter is 110m

If the land sells for \$50 per square metre <sup>area</sup>, then what is the value of this property?

①  $x$ : width     $y$ : length

②  $y = 2x + 10$

$$y = 2(15) + 10 = 40$$

$$2x + 2y = 110$$

$$30 + 80 = 110 \\ 110 = 110$$

③ solve

$$2x + 2(2x + 10) = 110$$

$$2x + 4x + 20 = 110$$

$$6x + 20 = 110$$

$$6x = 90$$

$$x = 15$$

④ Answer: a)  $A = x \cdot y$   
 $= 15 \cdot 40$   
 $= 600 \text{m}^2$

b)  $600 \text{m}^2 \times \$50/\text{m}^2 = \underline{\underline{\$30\,000}}$

Solve the systems.

$$1 \quad \left. \begin{array}{l} y = 5x + 9 \\ y = 3x - 12 \end{array} \right) \text{by comparison}$$

$$5x + 9 = 3x - 12$$

$$2x + 9 = -12$$

$$2x = -21$$

$$x = -10.5$$

$$y = 3(-10.5) - 12$$

$$= -31.5 - 12$$

$$= -43.5$$

$$\left( -10.5, -43.5 \right)$$

$$\text{by substitution} \left( \begin{array}{l} 2 \quad y = 4x - 1 \\ 3x - 2y + 11 = 0 \end{array} \right)$$

$$3x - 2(4x - 1) + 11 = 0$$

$$3x - 8x + 2 + 11 = 0$$

$$-5x + 13 = 0$$

$$-5x = -13$$

$$x = 2.6$$

$$y = 4(2.6) - 1$$

$$= 10.4 - 1$$

$$= 9.4$$

$$\left( 2.6, 9.4 \right)$$