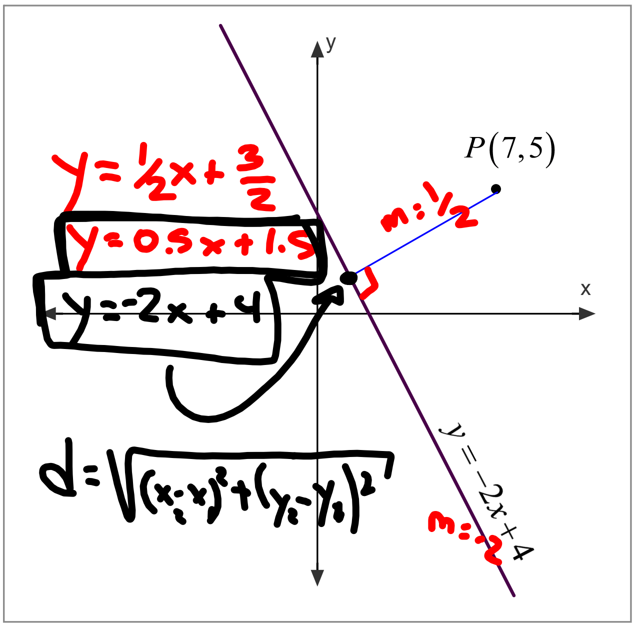
(Shortest) Distance from a Point to a Line



The shortest distance from a point to a line is the distance that runs perpendicular to that line.

Shortcut: There is a formula for calculating the distance.

The line needs to be in general form

$$\boxed{d(P,l)=\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}}$$

where x and y are the coordinates of the point.

$P(7,5)$ $\ell: y=-2x+4$

convert

$0=\boxed{-2x-y+4}$

$$d(P,\ell)=\frac{|-2x-1y+4|}{\sqrt{(-2)^2+(-1)^2}}$$

$$=\frac{|-2(7)-1(5)+4|}{\sqrt{4+1}}=\frac{|-14-5+4|}{\sqrt{5}}$$

$$=\frac{|-15|}{\sqrt{5}}=\frac{15}{\sqrt{5}}=6.71\,\underline{u}$$

Example: Determine the distance from the point $Z(3,11)$ to the line $\underbrace{\overset{A}{3}x - \overset{B}{4}y + \overset{C}{10}} = 0.$

$$d(Z,\ell) = \frac{|3(3) - 4(11) + 10|}{\sqrt{3^2 + (-4)^2}}$$

$$= \frac{|9 - 44 + 10|}{\sqrt{9+16}}$$

$$= \frac{|-25|}{+\sqrt{25}} = \frac{25}{5} = \underline{5 \text{ units}}$$

Example: Determine the distance from the point $Q(\overset{x}{-5},\overset{y}{4})$ to the line $\frac{x}{3}-\frac{y}{1}=1$.

$$3\cdot\left(\frac{x}{3}-\frac{y}{1}-1=0\right)$$

$$1x-3y-3=0$$

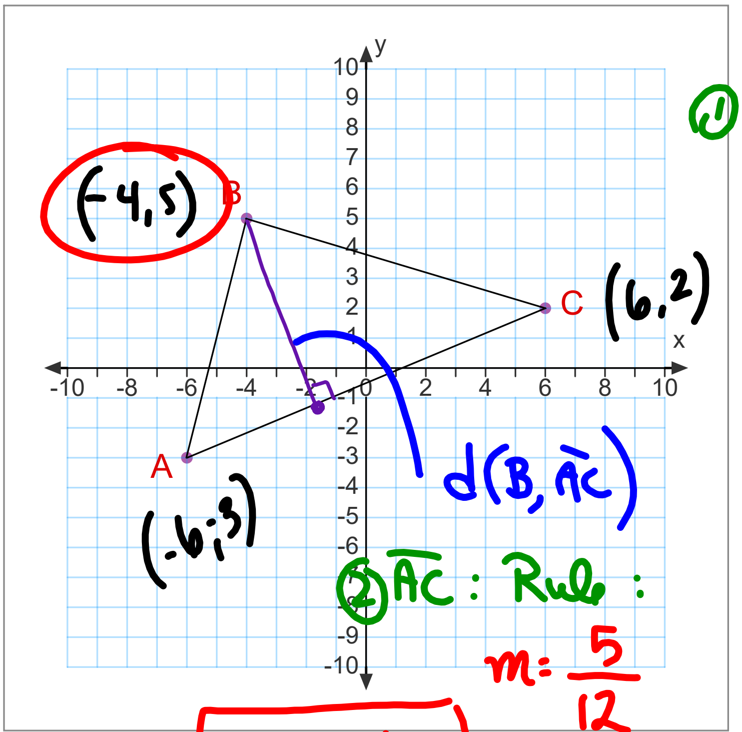
$$d(Q,\ell)=\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$$

$$=\frac{|-5-3(4)-3|}{\sqrt{1^2+(-3)^2}}$$

$$=\frac{|-5-12-3|}{\sqrt{1+9}}=\frac{|-20|}{\sqrt{10}}=\frac{20}{\sqrt{10}}=6.32 \text{ units}$$

Example: Determine the area of triangle ABC.

$A = \frac{b \cdot h}{2}$ — base — height



① base = $\overline{AC}$

$m\overline{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6--6)^2 + (2--3)^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

= 13 units

② $\overline{AC}$: Rule :

$m = \frac{5}{12}$

$y = \frac{5}{12}x + b \Rightarrow 2 = \frac{5}{12}(6) + b$

$-\frac{1}{2} = b \Leftarrow 2 = 2.5 + b$

$\boxed{y = \frac{5}{12}x - \frac{1}{2}}$

$\boxed{y = \frac{5}{12}x - \frac{1}{2}}$ $\Rightarrow$ general

$\left(0 = \frac{5}{12}x - y - \frac{1}{2}\right) \cdot 12$

$0 = 5x - 12y - 6$

$h = \frac{|5(-4) - 12(5) - 6|}{\sqrt{5^2 + (-12)^2}}$

$\frac{|-86|}{13} = \frac{86}{13}$

$\frac{b \cdot h}{2} = A$

$b = 13$

$h = \frac{86}{13}$

$\frac{\cancel{13} \cdot \frac{86}{\cancel{13}}}{2} = \frac{86}{2} = 43$