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11. The following functions have a rule of the form $f(x) = a(2)^{b(x-h)} + k$.

$f_1(x) = 3(2)^x$; $f_2(x) = 2^{3x}$; $f_3(x) = 2^{x+3}$; $f_4(x) = 2^x + 3$; $f_5(x) = -3(2)^{\frac{1}{2}(x+4)} - 5$;

Complete the table on the right by determining, for each function, the parameters a, b, h and k and by giving the rule of the transformation which enables you to obtain the graph of the function from the graph of the basic function $f(x) = 2^x$.

	a	b	h	k	Rule
f_1	3	1	0	0	$(x, y) \rightarrow$ **(x, 3y)**
f_2	1	3	0	0	$(x, y) \rightarrow \left(\frac{x}{3}, y\right)$
f_3	1	1	-3	0	$(x, y) \rightarrow$ **(x − 3, y)**
f_4	1	1	0	3	$(x, y) \rightarrow$ **(x, y + 3)**
f_5	-3	$\frac{1}{2}$	-4	-5	$(x, y) \rightarrow$ **(2x − 4, −3y − 5)**

12. In each of the following cases, a transformation is applied to the basic exponential function $y = 5^x$.

Find the rule of the function whose graph is obtained by the following transformations.

a) $(x, y) \rightarrow (x, -y)$ $y = -5^x$

b) $(x, y) \rightarrow (x - 2, y - 3)$ $y = 5^{x+2} - 3$

c) $(x, y) \rightarrow \left(\frac{x}{2}, y\right)$ $y = 5^{2x}$

d) $(x, y) \rightarrow (2x, y)$ $y = 5^{\frac{x}{2}}$

e) $(x, y) \rightarrow (3x, -2y)$ $y = -2(5)^{\frac{x}{3}}$

f) $(x, y) \rightarrow \left(\frac{x}{3} + 1, 2y - 1\right)$ $y = 2(5)^{3(x-1)} - 1$

13. Write the rules of the following exponential functions in the form $y = ac^{b(x-h)} + k$.

a) $y = 2^{3x-6}$ $y = 2^{3(x-2)}$

b) $y = 5^{-2x+6} + 1$ $y = 5^{-2(x-3)} + 1$

c) $y = 5(3)^{2x+1}$ $y = 5(3)^{2\left(x+\frac{1}{2}\right)}$

d) $y = 3\left(\frac{1}{2}\right)^{4-2x} - 5$ $y = 3\left(\frac{1}{2}\right)^{-2(x-2)} - 5$

$= 3\left(\frac{1}{2}\right)^{-2x+4} - 5$

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14. Solve the following exponential equations.

a) $3^x = 243$ $x = 5$

b) $2^x = \frac{1}{8}$ $x = -3$

c) $2(5)^x = 250$ $x = 3$

d) $5^{2x} - 1 = 0$ $x = 0$

e) $2(5)^x - 48 = 2$ $x = 2$

f) $9^x - 27 = 0$ $x = \frac{3}{2}$

g) $3(4)^x - 96 = 0$ $x = \frac{5}{2}$

h) $\frac{1}{2}(8)^x - 16 = 0$ $x = \frac{2}{3}$ 5 3

i) $27\left(\frac{4}{9}\right)^x - 8 = 0$ $x = \frac{3}{2}$

15. Determine the zero, if it exists, of the following exponential functions.

a) $y = 5(3)^{x-2} - 15$ — ***3***

b) $y = 2(3)^{-(x+2)} - 18$ — ***-4***

c) $y = -3\left(\frac{1}{2}\right)^{-2(x+3)} + 12$ — ***-2***

d) $y = -5\left(\frac{1}{5}\right)^{x-1} + 46$ — ***0*** 1.139

e) $y = -4\left(\frac{2}{3}\right)^{x-1} + 9$ — ***-1***

f) $y = 3\left(\frac{2}{5}\right)^{-2(x+1)} + \frac{12}{25}$ — ***No zero***

16. Solve the following exponential equations.

a) $2^{3x} \cdot 2^{2x} = \frac{1}{4}$ — $x = -\frac{2}{5}$

b) $\frac{3^x}{3^{2x}} = 27$ — ***x = -3***

c) $\left(\frac{1}{2}\right)^x = 16$ — ***x = -4***

d) $2^x \cdot 2^x = 64$ — ***x = 3***

e) $(2^x)^2 = 16(2)^x$ — ***x = 4***

f) $2^{x^2} = 16$ — ***x = -2 or x = 2***

17. The growth of a herd of bison follows the rule $P(t) = P_0 \times 2^{\frac{t}{10}}$ where P_0 represents the initial population and $P(t)$ the population after t years. In how many years will the bison population quadruple its initial population?

$4P_0 = P_0(2)^{\frac{t}{10}} \Leftrightarrow 4 = 2^{\frac{t}{10}} \Leftrightarrow t = 20$. After 20 years.

18. A mosquito population doubles every seven days. If there were 5 mosquitoes initially, after how many days will the population contain 80 mosquitoes?

$5(2)^{\frac{t}{7}} = 80 \Leftrightarrow 2^{\frac{t}{7}} = 16 \Leftrightarrow t = 28$. After 28 days.

19. A 100 g radioactive mass disintegrates according to the rule $m(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{4}}$ where $m(t)$ is the resulting mass after t hours.

a) Determine after how many hours the resulting mass is equal to 25 g. ***8 hours***

b) We call the **half-life** of a radioactive substance the time necessary for its mass to be reduced by half by disintegration. What is the half-life of this mass? ***4 hours***