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11. The following functions have a rule of the form $f(x) = a(2)^{b(x-h)} + k$.

$$f_1(x) = 3(2)^x$$
; $f_2(x) = 2^{3x}$; $f_3(x) = 2^{x+3}$; $f_4(x) = 2^x + 3$; $f_5(x) = -3(2)^{\frac{1}{2}(x+4)} - 5$;

Complete the table on the right by determining, for each function, the parameters a, b, h and k and by giving the rule of the transformation which enables you to obtain the graph of the function from the graph of the basic function $f(x) = 2^x$.

	a	b	h	k	Rule :
f_1	3	1	0	0	$(x,y) \rightarrow (x,3y)$
f_2	1	3	0	0	$(x,y) o \left(\frac{x}{3},y\right)$
f_3	1	1	-3	0	$(x,y) \xrightarrow{\cdot} (x-3,y)$
f_4	1	1	0	3	$(x,y) \rightarrow (x,y+3)$
f_5	-3	1/2	-4	-5	$(x,y) \to (2x-4, -3y-5)$

12. In each of the following cases, a transformation is applied to the basic exponential function $y = 5^x$.

Find the rule of the function whose graph is obtained by the following transformations.

a)
$$(x, y) \to (x, -y)$$
 $y = -5x$

b)
$$(x, y) \rightarrow (x - 2, y - 3)$$
 $y = 5^{x+2} - 3$

c)
$$(x, y) \rightarrow \left(\frac{x}{2}, y\right) \quad y = 5^{2x}$$

d)
$$(x, y) \to (2x, y)$$
 $y = 5^{\frac{x}{2}}$

e)
$$(x, y) \to (3x + 2y)$$
 $y = -2(5)^{\frac{x}{3}}$

f)
$$(x,y) \to \left(\frac{x}{3}+1,2y-1\right)$$
 $y = 2(5)^{3(x-1)}-1$

13. Write the rules of the following exponential functions in the form $y = ac^{b(x-h)} + k$.

a)
$$y = 2^{3x-6}$$
 $y = 2^{3(x-2)}$

b)
$$y = 5^{-2x+6} + 1$$
 $y = 5^{-2(x-3)} + 1$

c)
$$y = 5(3)^{2x+1}$$
 $y = 5(3)^{2(x+\frac{1}{2})}$

d)
$$y = 3\left(\frac{1}{2}\right)^{4-2x} - 5$$
 $y = 3\left(\frac{1}{2}\right)^{-2(x-2)} - 5$

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14. Solve the following exponential equations.

a)
$$3^x = 243$$

b)
$$2^x = \frac{1}{8}$$

c)
$$2(5)^x = 250$$

$$x = 5$$

$$x = -3$$

$$x = 3$$

d)
$$5^{2x} - 1 = 0$$
 $x = 0$

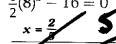
e)
$$2(5)^x - 48 = 2$$

$$9^x - 27 = 0$$

g)
$$3(4)^x - 96 = 0$$

i)
$$27(\frac{4}{9})^x - 8 = 0$$

$$x = \frac{5}{2}$$





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15. Determine the zero, if it exist	ts, of the following exponenti	al functions.
a) $y = 5(3)^{x-2} - 15$	b) $y = 2(3)^{-(x+2)} - 18$	c) $y = -3\left(\frac{1}{2}\right)^{-2(x+3)} + 12$
3	-4	
d) $y = -5\left(\frac{1}{5}\right)^{x-1} + 25$	e) $y = -4\left(\frac{2}{3}\right)^{x-1} + 9$	f) $y = 3\left(\frac{2}{5}\right)^{-2(x+1)} + \frac{12}{25}$
(0) 1, 139	-1	No zero

a)
$$y = 5(3)^{x-2} - 15$$

b)
$$y = 2(3)^{-(x+2)} - 18$$

c)
$$y = -3\left(\frac{1}{2}\right)^{-2(x+3)} + 12$$

$$-(1)^{x-1}$$

$$y = -5\left(\frac{1}{5}\right)^{1/2} + 45$$

e)
$$y = -4\left(\frac{2}{3}\right)^{x-1} + 9$$

f)
$$y = 3\left(\frac{2}{5}\right)^{-2(x+1)} + \frac{12}{25}$$

a)
$$2^{3x} \cdot 2^{2x} = \frac{1}{4}$$

b)
$$\frac{3^x}{3^{2x}} = 27$$

c)
$$\left(\frac{1}{2}\right)^x = 16$$

d)
$$2^x \cdot 2^x = 64$$

x = 3

e)
$$(2^x)^2 = 16(2)^x$$

f)
$$2^{x^2} = 16$$

 $x = -2 \text{ or } x = 2$

17. The growth of a herd of bison follows the rule
$$P(t) = P_0 \times 2^{\frac{t}{10}}$$
 where P_0 represents the initial population and $P(t)$ the population after t years. In how many years will the bison population quadruple its initial population?

$$4 P_0 = P_0(2)^{\frac{t}{10}} \Leftrightarrow 4 = 2^{\frac{t}{10}} \Leftrightarrow t = 20$$
. After 20 years.

$$5(2)^{\frac{t}{7}} = 80 \Leftrightarrow 2^{\frac{t}{7}} = 16 \Leftrightarrow t = 28$$
. After 28 days.

19. A 100 g radioactive mass disintegrates according to the rule
$$m(t) = 100 \left(\frac{1}{2}\right)^{\frac{t}{4}}$$
 where $m(t)$ is the resulting mass after t hours.

- a) Determine after how many hours the resulting mass is equal to 25 g. _
- b) We call the half-life of a radioactive substance the time necessary for its mass to be reduced by half by disintegration. What is the half-life of this mass? 4 hours