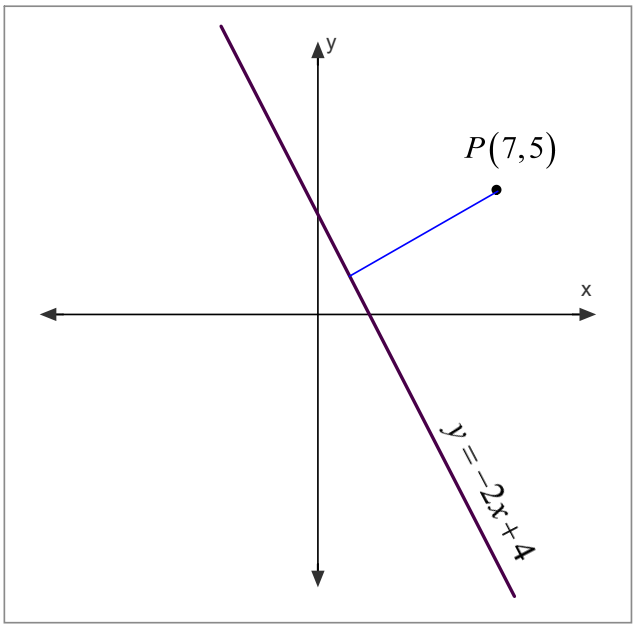
Distance from a Point to a Line



The shortest distance from a point to a line is the distance that runs perpendicular to that line.

Shortcut: There is a formula for calculating the distance.

The line needs to be in general form

$$d(P,l)=\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$$

where x and y are the coordinates of the point.

Example: Determine the distance from the point $Z(3,11)$ to the line $3x-4y+10=0$.

$x \quad y$

$$d(Z,\ell) = \frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$$

$$= \frac{|3x-4y+10|}{\sqrt{3^2+(-4)^2}}$$

$$= \frac{|3(3)-4(11)+10|}{\sqrt{9+16}}$$

$$= \frac{|9-44+10|}{\sqrt{25}}$$

$$\therefore \frac{|-25|}{5} = \frac{25}{5}$$

$$= 5 \text{ units}$$

Example: Determine the distance from the point $P(7,5)$ to the line $y = -2x + 4$.

① $0 = -2x - y + 4$

② $d(P,\ell) = \dfrac{|-2(7) - (5) + 4|}{\sqrt{(-2)^2 + (-1)^2}}$

$= \dfrac{|-14 - 5 + 4|}{\sqrt{4+1}}$

$= \dfrac{|-15|}{\sqrt{5}}$

$6.71u = \dfrac{15}{\sqrt{5}}$

Example: Determine the distance from the point $Q(-5,4)$ to the line $\frac{x}{3}-\frac{y}{1}=1$.

$\frac{x}{3}-\frac{y}{1}=1$

$3\left(\frac{x}{3}-\frac{y}{1}-1=0\right)$

$1x-3y-3=0$

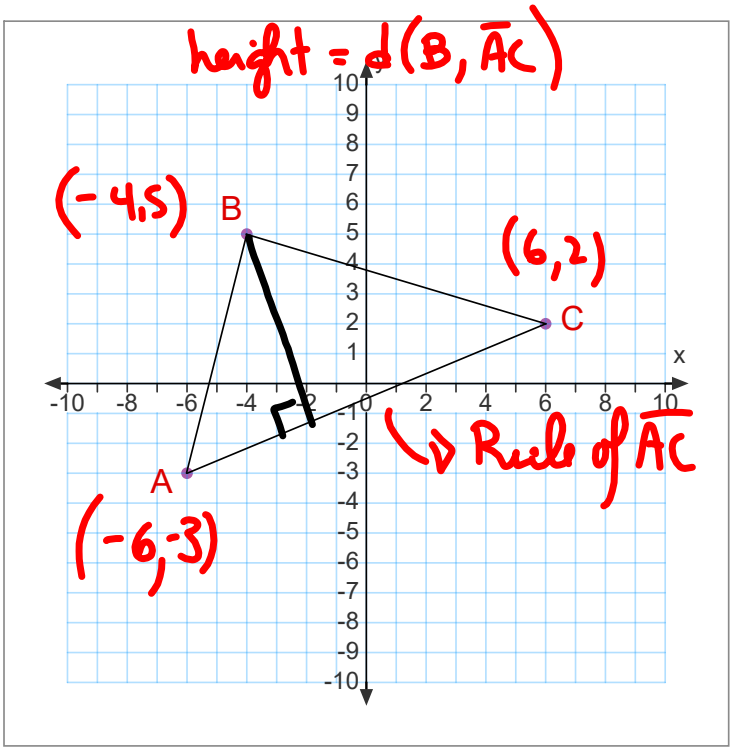
$d(Q,\ell)=\frac{|1(-5)-3(4)-3|}{\sqrt{1^2+(-3)^2}}$

$=\frac{|-5-12-3|}{\sqrt{1+9}}$

$=\frac{|-20|}{\sqrt{10}}$

$6.32 \text{ units} = \frac{20}{\sqrt{10}}$

Example: Determine the area of triangle ABC.



$A = \frac{b \cdot h}{2}$ base : $\overline{AC}$

base = $\sqrt{(6-(-6))^2 + (2-(-3))^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25}$

$= \sqrt{169} = 13$