Linear Function

or

First-Degree Polynomial Function

A linear function has a graph that is a straight line.

There are two things most often referred to regarding a linear function: its slope and y-intercept.

y-intercept: or initial value - Where the line intersects the y-axis

- The value of y when $x = 0$.

Slope: or rate of change - The line either increases or decreases.

- There is a pattern: as the x-values change, the y-values go up or down at a constant rate.

- The slope/rate of change compares how much the dependent variable changes as a result of a change in the independent variable.

Slope:

$$m = \frac{\Delta y}{\Delta x} \quad \text{or} \quad m = \frac{rise}{run}$$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

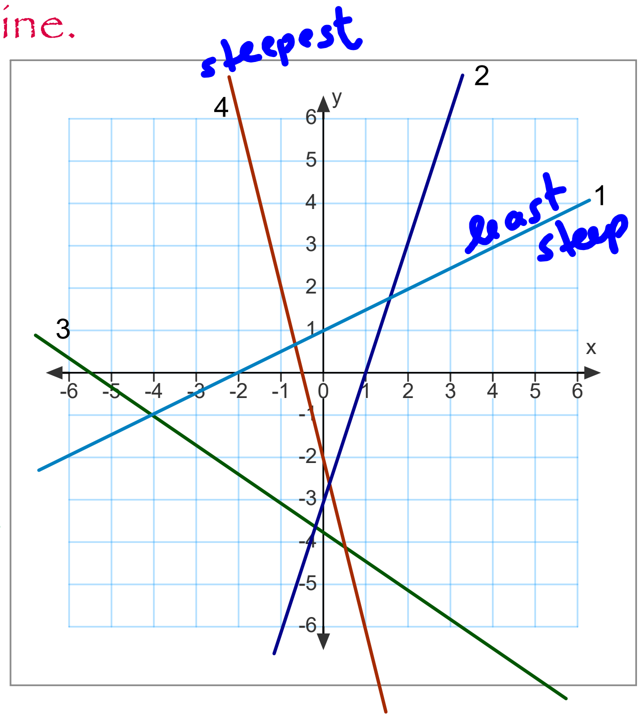
The value of m indicates the steepness of the line. The bigger $|m|$ is, the steeper the line.

1) $m = \frac{1}{2}$

2) $m = \frac{3}{1}$

3) $m = -\frac{2}{3}$

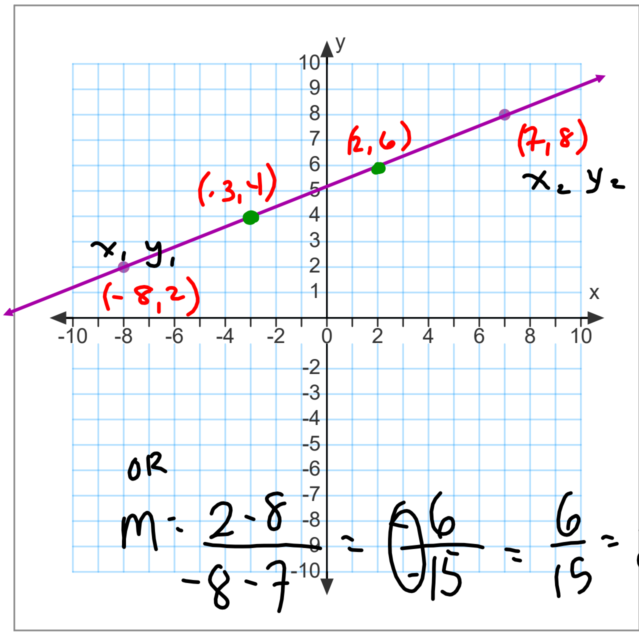
4) $m = -\frac{4}{1}$



If m is positive, then the line is increasing.

If m is negative, then the line is decreasing.

What is the slope of $\overleftrightarrow{AB}$?



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{7-(-8)} = \frac{6}{15} = \frac{2}{5}$$

OR

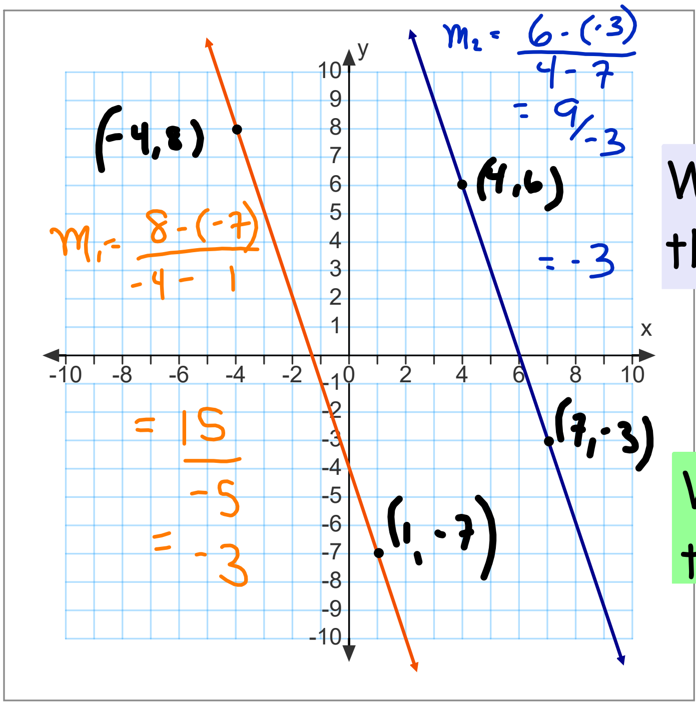
$$m = \frac{2-8}{-8-7} = \frac{-6}{-15} = \frac{6}{15} = \frac{2}{5}$$

What is the slope of the line that passes through points $Q(\overset{x_1}{-4},\overset{y_1}{6})$ and $R(\overset{x_2}{11},\overset{y_2}{-1})$?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-6}{11-(-4)} = \frac{-7}{15}$$

Special Lines - Special Slopes

Determine the slopes of the two lines shown below.



What do you notice about the slopes?

They are the same.

What is special about these lines?

They are parallel.

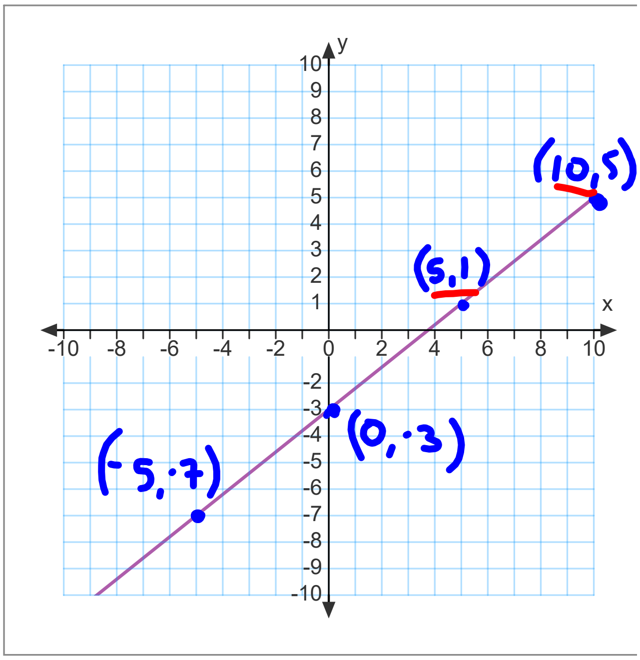
Distinct Parallel lines have the **same slope**, **but different points.**

What is the slope of this line?

$m = \frac{5-1}{10-5} = \frac{4}{5}$

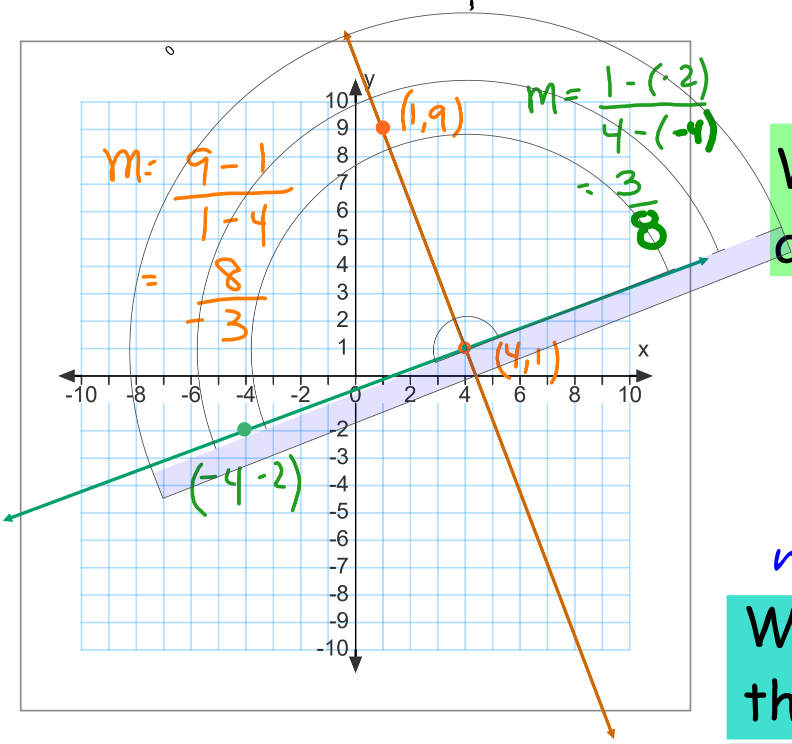
$m = \frac{8}{10}$ $m = 0.8$

$m = \frac{40}{50}$ $m = \frac{80}{100}$



Coincident Parallel lines have the **same slope and all the same points**.

Determine the slopes of the two lines shown below.



What do you notice about the slopes?

They are opposites: opposite sign and reciprocals

negative reciprocals

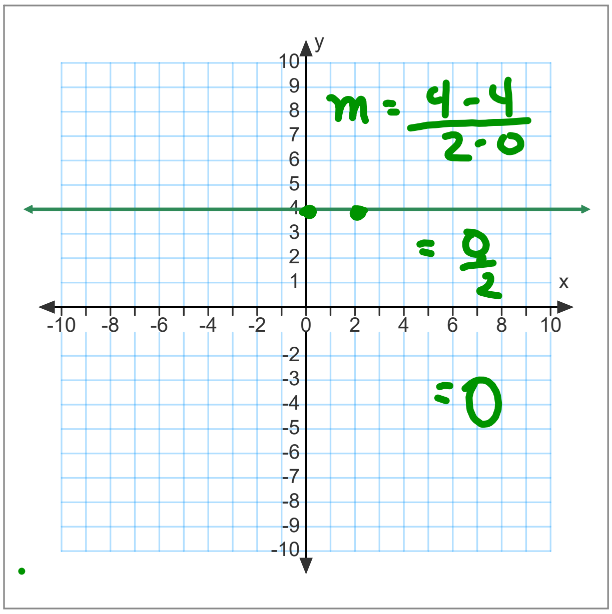
What is special about these lines?

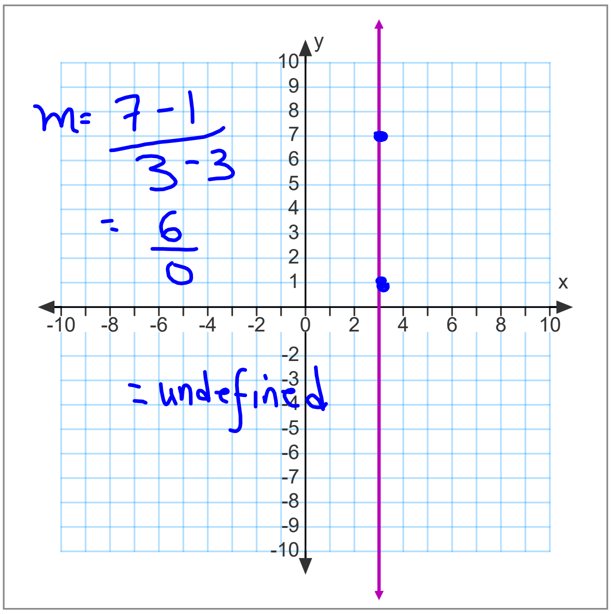
They are perpendicular

The slopes of perpendicular lines are negative reciprocals.

Also : $m_1 \times m_2 = -1$

Determine the slopes of the following lines.





A Horizontal line's slope is equal to 0.

A Vertical line's slope is undefined.

Basic Linear Function

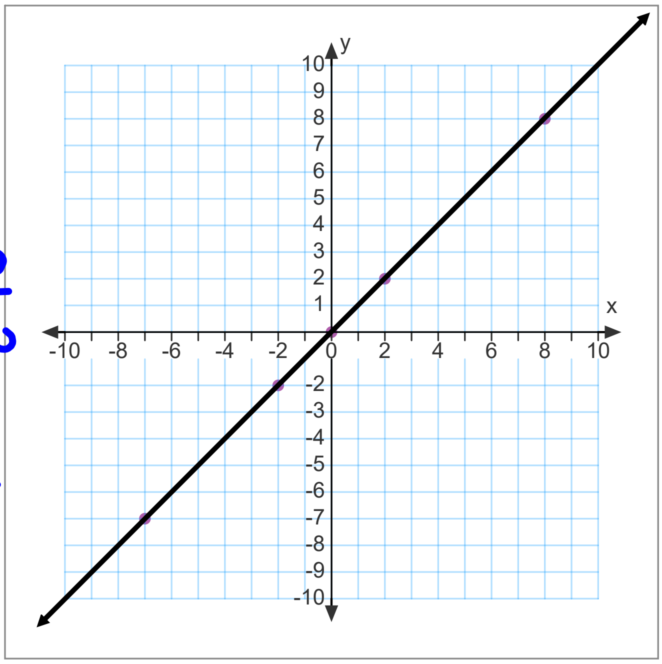
Rule: $y = x$

x	y
2	2
-7	-7
0	0
-2	-2
8	8

$m = \frac{2-0}{2-0}$

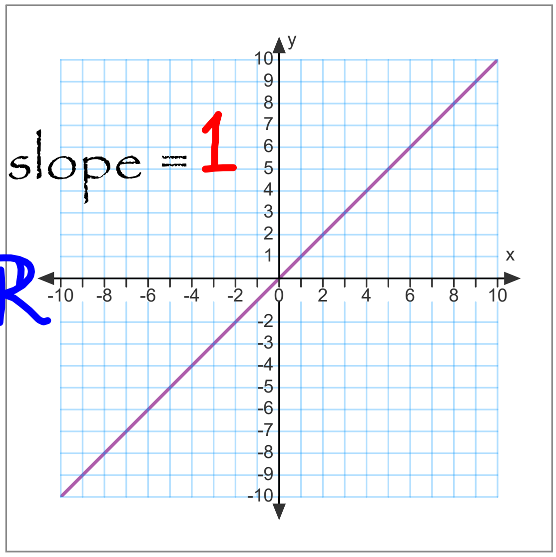
$= \frac{2}{2}$

$= 1$



$y = x$

Properties:



slope = 1

Dom: $\mathbb{R}$ Ran: $\mathbb{R}$

Variation: Increasing over $\mathbb{R}$

Sign: Positive : $[0, +\infty[$

Negative : $]-\infty, 0]$

Intercepts: x-int = 0

y-int = 0

Extrema: None

Transformed Function

When change the y-intercept and the slope we get new linear functions.

We use the following equation:

independent variable

$$y = mx + b$$

dependent variable

where m is the slope or rate of change

and b is the y-intercept or initial value

This form of a line is known as standard or functional form.

Which means: $f(x) = mx + b$

Find the Rule

- Given two points

Example: Determine the equation of the line that passes through the points *A*(-2, 7) and *B*(2, 9).

$$\cdot\ y = mx + b$$

Step 1: Determine the slope (m)

A(-2, 7) B(2, 9)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{9 - 7}{2 - (-2)}$$

$$= \frac{2}{4} \text{ or } 0.5 \text{ or } \frac{1}{2}$$

$\therefore y = \frac{1}{2}x + \underline{\underline{b}}$

Step 2: Determine the value of b, the initial value.

-- Choose one of the points, A or B.

A(-2, 7) B(2, 9)

A: x = -2 and y = 7

B: x = 2 and y = 9

-- Fill in x, y and m in the equation and solve for b.

$$y = mx + b$$

$$y = 0.5x + b$$

using B(2,9) ⇒ $9 = 0.5(2) + b$

$$9 = 1 + b$$

$$-1 \qquad -1$$

$$8 = b$$

Step 3: Put your m and b into the equation.

$$y = 0.5x + 8$$

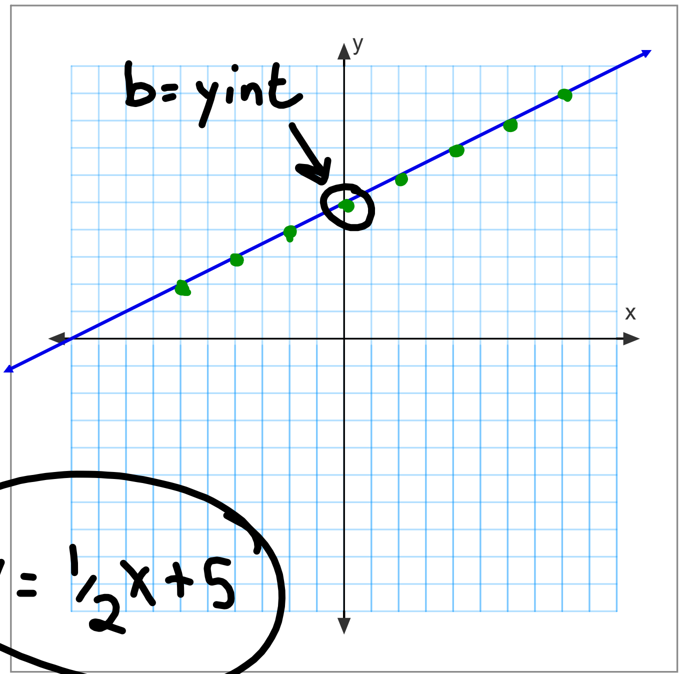
Example: Determine the equation of the line shown below.

using: (2,6)
(-4, 3)

$m = \frac{3-6}{-4-2} = \frac{-3}{-6} = \frac{1}{2}$

$m = \frac{6-3}{2-(-4)} = \frac{3}{6}$

$b = 5$



$y = \frac{1}{2}x + 5$

Example: Determine the equation of the line that passes through points $K(1,12)$ and $L(9,2)$.

②

① $m = \frac{2-12}{9-1} = \frac{-10}{8} = -\frac{5}{4} = -1.25$

② $2 = -\frac{5}{4}(9) + b$

$2 = -\frac{45}{4} + b$

$\frac{2}{1} + \frac{45}{4} = b$

$\frac{8}{4} + \frac{45}{4} = b$

$\frac{53}{4}$

$y = -\frac{5}{4}x + \frac{53}{4}$

$y = -1.25x + 13.25$