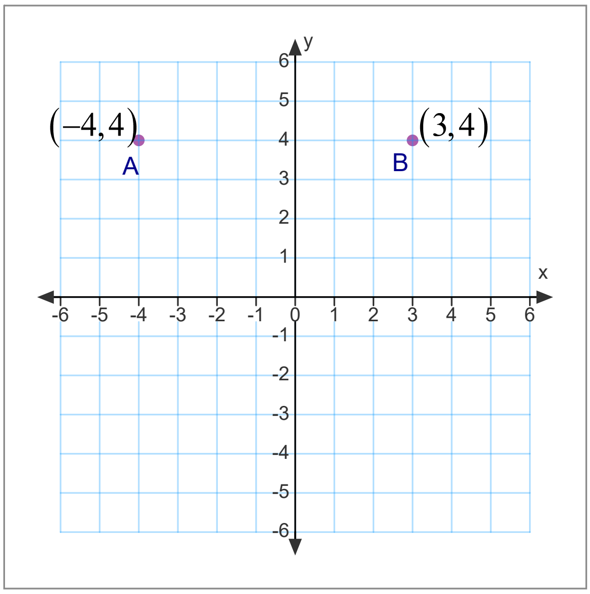
Distance Between Two Points

What is the distance from A to B?

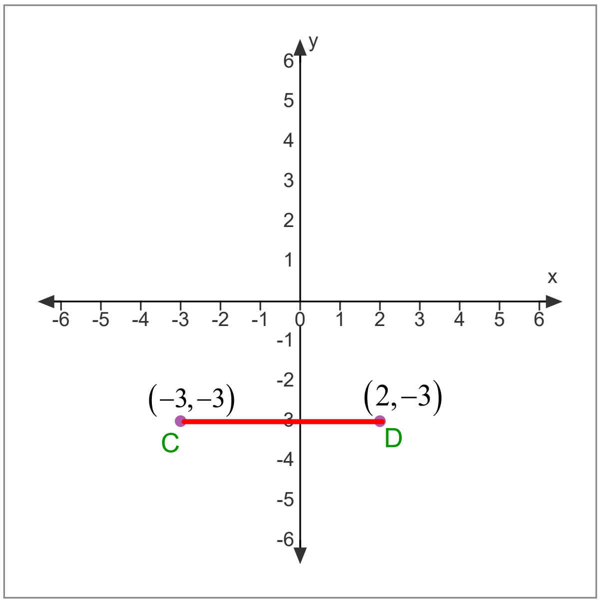


$d(A,B) = \boxed{7}$ units

Example:

$m\overline{CD} = ?$

5 units



What is the distance between points $M(-4,-3)$ and $N(5,-3)$?

$d(M,N) = 9$

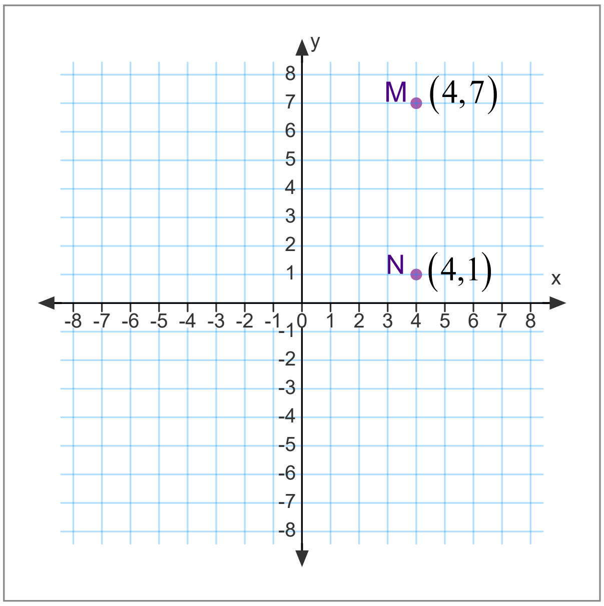
Points that have the same y coordinates are in line horizontally, so the distance between these two points is.....

The horizontal distance between two points is the difference between the x co-ordinates: $(x_2 - x_1)$

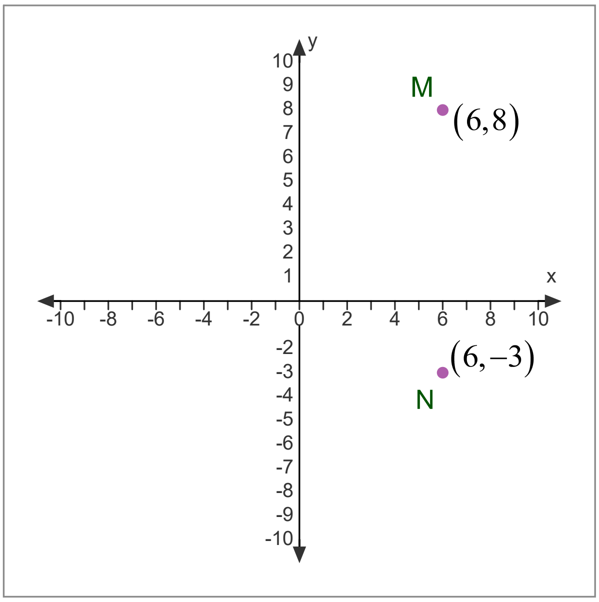
$|(x_2 - x_1)|$

What is the distance from M to N?

$$d(M,N) = \boxed{6} \text{ units}$$



$$d(M,N)= \boxed{11} \text{ units}$$



What is the distance between points $P(2,17)$ and $Q(2,9)$?

$d(P,Q) = 8$

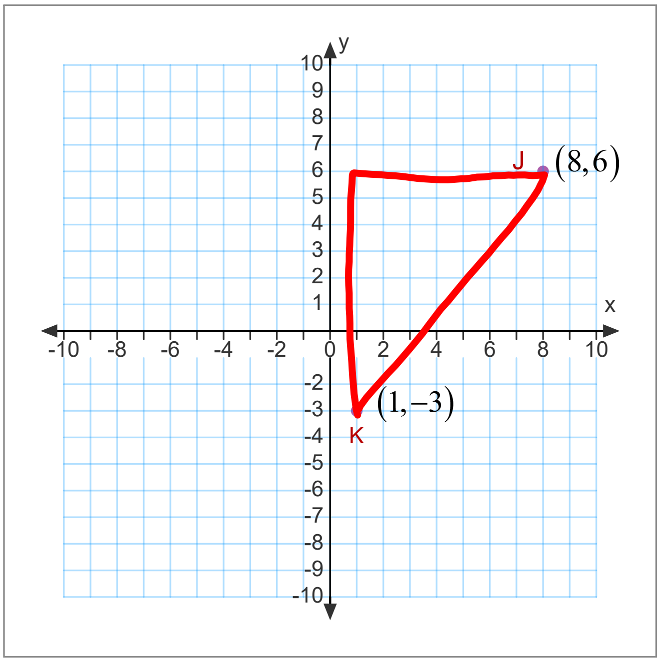
Points that have the same x coordinates are in line vertically, so the distance between these two points is....

The <u>vertical distance</u> between two points is the <u>difference of the y co-ordinates</u>: $(y_2 - y_1)$

$$|(y_2 - y_1)|$$

For Oblique (slanted) distances:

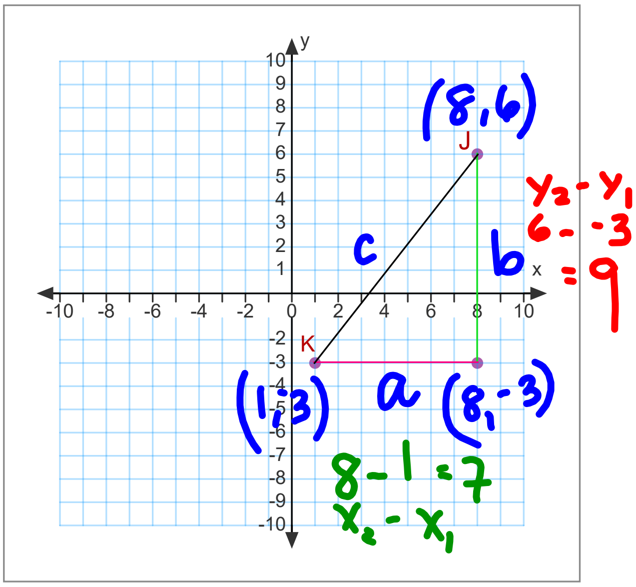
Find the distance from J to K.



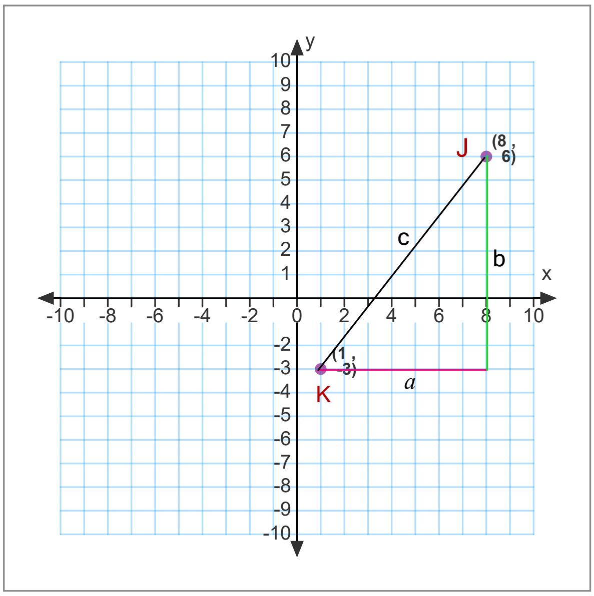
Hint:

Pythagoras' Theorem

On the Cartesian plane, we can make a right triangle whose hypotenuse is the segment JK.



By calculating the vertical and horizontal distances (the lengths of the legs of the triangle) we can use Pythagoras' theorem to find $d(J, K)$.



Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

$a = 7$

$b = 9$

$c = ?$

$$7^2 + 9^2 = c^2$$

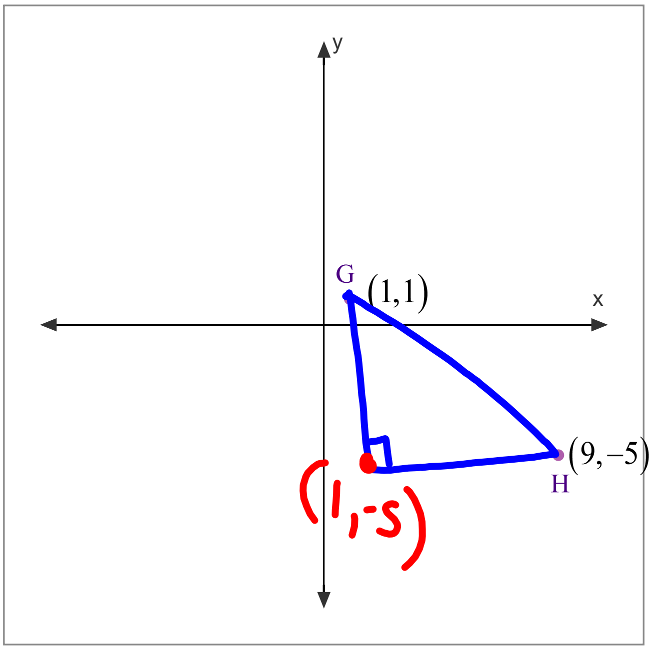
$$49 + 81 = c^2$$

$$130 = c^2$$

$$\sqrt{130} = c$$

$$11.4 \doteq c \qquad \therefore\ d(J, K) = 11.4\ units$$

Calculate $d(G, H)$



1. Find the horizontal distance $(x_2 - x_1)$.

2. Find the vertical distance $(y_2 - y_1)$.

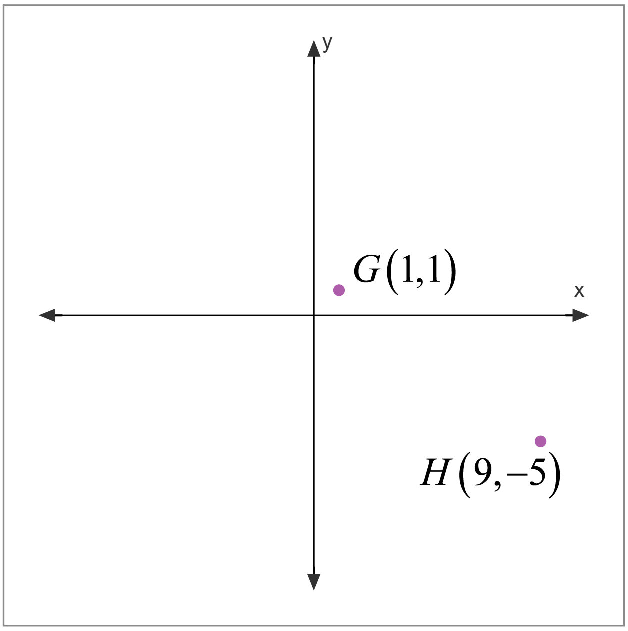
3. Calculate the distance between the points using Pythagoras' theorem.

These steps can be condensed into a single formula called the ...

Distance Formula. $A(x_1, y_1)$ $B(x_2, y_2)$

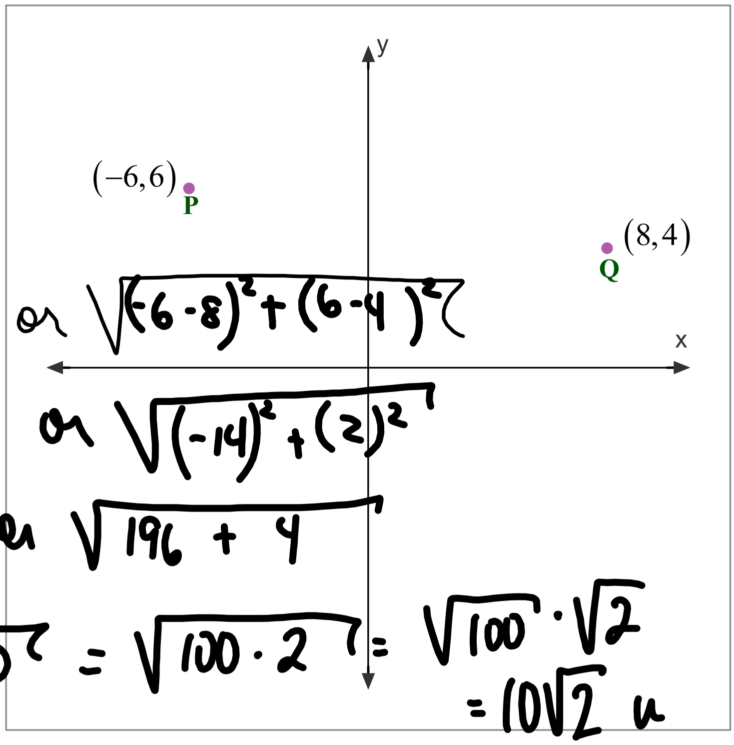
$$\boxed{d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$
\begin{aligned}
d(G,H) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(9-1)^2 + (-5-1)^2} \\
&= \sqrt{8^2 + (-6)^2} \\
&= \sqrt{64+36} \\
&= \sqrt{100} \\
&= 10
\end{aligned}
$$



$d(G,H) = 10 \textit{ units}$

Calculate $d(P, Q)$



$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(8--6)^2+(4-6)^2}$ or $\sqrt{(-6-8)^2+(6-4)^2}$

$= \sqrt{(14)^2+(-2)^2}$ or $\sqrt{(-14)^2+(2)^2}$

$= \sqrt{196+4}$ or $\sqrt{196+4}$

$= \sqrt{200} = \sqrt{100\cdot 2} = \sqrt{100}\cdot\sqrt{2}$

$= 10\sqrt{2}$ u

OR 14.14 u

Example: Calculate $m\overline{AB}$.

Note: $m\overline{AB} = d(A, B)$

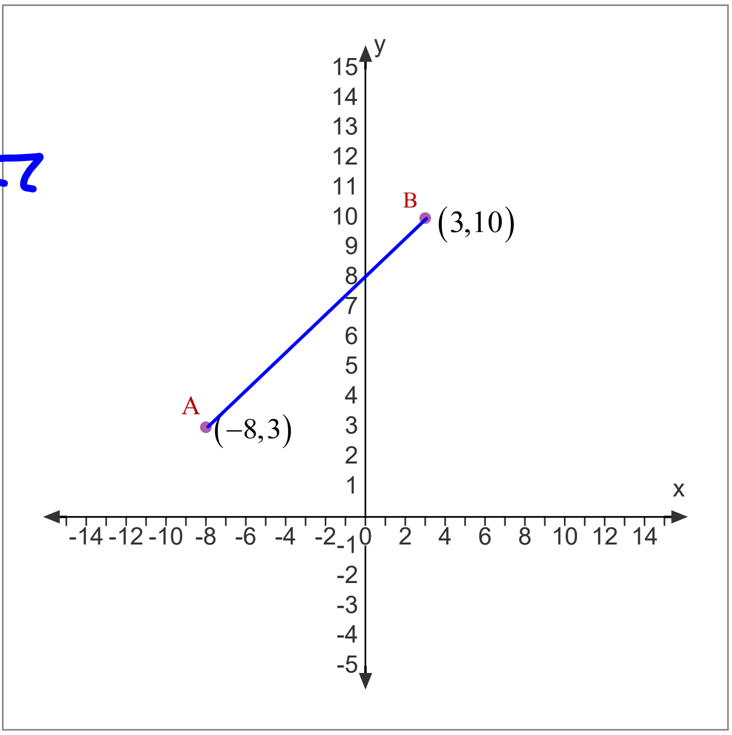
$= \sqrt{(3 - -8)^2 + (10 - 3)^2}$

$= \sqrt{11^2 + 7^2}$

$= \sqrt{121 + 49}$

$= \sqrt{170}$

$\approx$ 13.04 units



Example: Calculate $m\overline{FL}$, given $F(6,4)$ and $L(3,-6)$.

$$d(F,L) = \sqrt{(6-3)^2 + (4 - {}^-6)^2}$$
$$= \sqrt{3^2 + 10^2}$$
$$= \sqrt{9 + 100}$$
$$= \sqrt{109} \text{ or } 10.4$$

P 133 #6 Points $A(-4,1), B(1,6)$ & $C(1,1)$

$\triangle ABC \Rightarrow$ show that $\triangle ABC$ is a right isosceles triangle.

$d(A,B)$

$= \sqrt{(1--4)^2 + (6-1)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$ or $5\sqrt{2}$

$= 7.07$

$\sqrt{25 \cdot 2}$

$\sqrt{25} \cdot \sqrt{2}$

$d(B,C)$

$y_2 - y_1$

$6 - 1$

5

$d(A,C)$

$x_2 - x_1$

$1 - -4 = 5$

$m\overline{BC} = m\overline{AC} = 5$ ✓

$\therefore$ isosceles

$a^2 + b^2 = c^2$

$5^2 + 5^2 = (\sqrt{50})^2 \Rightarrow 25 + 25 = 50$

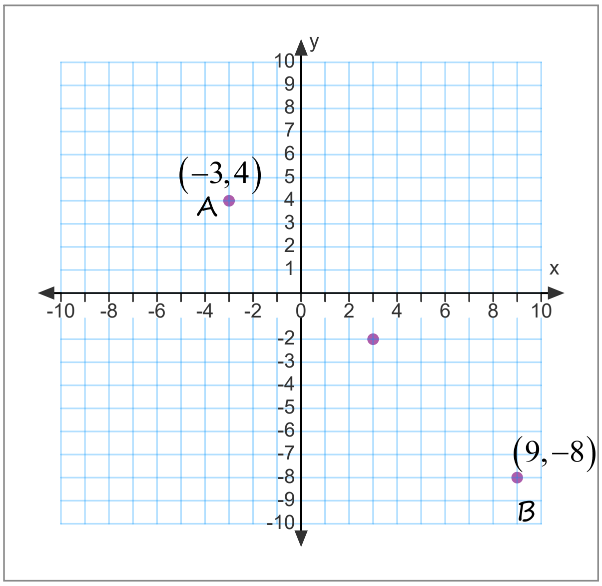
$50 = 50$ ✓

$\therefore$ right

Midpoint Formula

The midpoint is a point that lies exactly halfway between two other points.

Example: What are the co-ordinates of the point that lies half way between the points A(-3, 4) and B(9, -8)?



If we <u>average</u> the x co-ordinates and the y co-ordinates of the two points, then we get the x & y values that lie in the middle.

Let $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (9, -8)$

1. Average of x-values

$$\frac{-3+9}{2} = \frac{6}{2} = 3$$

2. Average of y-values

$$\frac{4+-8}{2} = \frac{-4}{2} = -2$$

Therefore, the point that lies halfway between A and B (the midpoint) is ...

$$\boxed{M(3,-2)}$$

average x average y

Midpoint Formula

$$x \text{ co-ordinate} = \frac{x_1 + x_2}{2}$$

$$y \text{ co-ordinate} = \frac{y_1 + y_2}{2}$$

$$\left.\begin{array}{l} x \text{ co-ordinate} = \dfrac{x_1 + x_2}{2} \\[2em] y \text{ co-ordinate} = \dfrac{y_1 + y_2}{2} \end{array}\right\} M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$