Radicals

Radicals are expressions that involve a root sign.

$\sqrt{n}$ is called a radical.

radical sign $\sqrt{n}$ radicand

Addition and Subtraction

Adding and subtracting radicals is like algebra - they have to be "like terms"; that is, the radicals must match.

Examples:

$$6\sqrt{7}-4\sqrt{7}=2\sqrt{7}$$

$$5\sqrt{2}+3\sqrt{2}=8\sqrt{2}$$

$$\sqrt{3}+2\sqrt{5}+10\sqrt{5}=\sqrt{3}+12\sqrt{5}$$

Multiplication and Division

Properties: 1) $\sqrt{m}\times\sqrt{n} \Leftrightarrow \sqrt{m\times n}$ 2) $\frac{\sqrt{m}}{\sqrt{n}} \Leftrightarrow \sqrt{\frac{m}{n}}$

Example: $4\sqrt{2}\times 3\sqrt{6}$

Multiply/divide the coefficients and multiply/divide the radicands. Like terms are <u>not</u> necessary.

$12\sqrt{4\cdot 3}$

$12\cdot\sqrt{4}\cdot\sqrt{3}$

$12\cdot 2\cdot\sqrt{3}$

$$4\sqrt{2}\times 3\sqrt{6} = (4\times 3)\sqrt{2\times 6}$$
$$= 12\sqrt{12} \Rightarrow 24\sqrt{3}$$

Examples:

$$5\sqrt{20} \div 3\sqrt{10} = (5 \div 3)\sqrt{20 \div 10}$$

$$= \frac{5}{3}\sqrt{2}$$

$$\underbrace{2\sqrt{3} \times 6\sqrt{8}}_{} \div 4\sqrt{12} = 3\sqrt{2}$$

$$12\sqrt{3 \cdot 8} \div 4\sqrt{12}$$

$$12\sqrt{24} \div 4\sqrt{12}$$

$$3\sqrt{24 \div 12}$$

Simplifying Radicals

Many radicals can be simplified.

Example: $\sqrt{27}$

1. Break down the radicand into two factors - one of which is a perfect square.

$\sqrt{27} = \sqrt{9 \cdot 3}$

2. Apply the multiplication property of radicals to create a coefficient times a radical.

$\sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3 \cdot \sqrt{3} = 3\sqrt{3}$

Example: $3\sqrt{72}$

$3\cdot\sqrt{9\cdot 8}$

$3\cdot\sqrt{9}\cdot\sqrt{8}$

$3\cdot 3\cdot\sqrt{8}$

$9\sqrt{8}$

$9\sqrt{4\cdot 2}$

$9\cdot\sqrt{4}\cdot\sqrt{2}$

$9\cdot 2\cdot\sqrt{2}$

$18\sqrt{2}$

or $3\sqrt{36\cdot 2}$

$3\sqrt{36}\cdot\sqrt{2}$

$3\cdot 6\sqrt{2}$

Example: $4\sqrt{48}+2\sqrt{12}$

$4\sqrt{16\cdot 3} + 2\sqrt{4\cdot 3}$

$4\cdot\sqrt{16}\cdot\sqrt{3} + 2\sqrt{4}\cdot\sqrt{3}$

$4\cdot 4\cdot\sqrt{3} + 2\cdot 2\cdot\sqrt{3}$

$16\sqrt{3} + 4\sqrt{3}$

$20\sqrt{3}$

Try these!

1. $3\sqrt{5}\times\left(-2\sqrt{3}\right)$

2. $2\sqrt{3}\left(5\sqrt{3}+\sqrt{5}\right)$

3. $\left(4\sqrt{5}+3\right)\left(4\sqrt{5}-3\right)$

4. $12\sqrt{30}\div 4\sqrt{5}$

5. $2\sqrt{75}-2\sqrt{108}+5\sqrt{75}-\sqrt{108}+3\sqrt{12}$

Simplify

1. $3\sqrt{5} \times -2\sqrt{3} = -6\sqrt{15}$

2. $2\sqrt{3}\left(5\sqrt{3} + \sqrt{5}\right) = 2\sqrt{3}\cdot 5\sqrt{3} + 2\sqrt{3}\cdot\sqrt{5}$
$= 10\sqrt{9} + 2\sqrt{15}$
$= 10\cdot 3 + 2\sqrt{15}$
$= 30 + 2\sqrt{15}$

3. $(4\sqrt{5} + 3)(4\sqrt{5} - 3) = 4\sqrt{5}\cdot 4\sqrt{5} + 4\sqrt{5}\cdot(-3) + 3\cdot 4\sqrt{5} + 3\cdot(-3)$
$= 16\sqrt{25} - 12\sqrt{5} + 12\sqrt{5} - 9$
$= 16(5) - 9$
$= 80 - 9$
$= 71$

4. $2\sqrt{75} - 2\sqrt{108} + 5\sqrt{75} - \sqrt{108} + 3\sqrt{12}$
$= 7\sqrt{75} - 3\sqrt{108} + 3\sqrt{12}$
$= 7\sqrt{25\cdot 3} - 3\sqrt{36\cdot 3} + 3\sqrt{4\cdot 3} = 35\sqrt{3} - 18\sqrt{3} + 6\sqrt{3}$
$= 23\sqrt{3}$

5. $12\sqrt{30} \div 4\sqrt{5} = \dfrac{12\sqrt{30}}{4\sqrt{5}}$
$= 3\sqrt{\dfrac{30}{5}}$
$= 3\sqrt{6}$

Work Book

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Questions 3, 4, 5, 6, 7 & 9

Text Book 1

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Questions 1, 2 & 3