Radicals

Radicals are expressions that involve a root sign.

$\sqrt{n}$ is called a radical.

radical sign $\sqrt{n}$ radicand

Addition and Subtraction

Adding and subtracting radicals is like algebra - they have to be "like terms" - the radicals must match.

Examples: $6\sqrt{7} - 4\sqrt{7} = 2\sqrt{7}$

$6\sqrt{3} - 2\sqrt{5} = 6\sqrt{3} - 2\sqrt{5}$

$5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$

$\sqrt{3} + 2\sqrt{5} + 10\sqrt{5} =$

Multiplication and Division

Properties: 1) $\sqrt{m}\times\sqrt{n} \Leftrightarrow \sqrt{m\times n}$ 2) $\frac{\sqrt{m}}{\sqrt{n}} \Leftrightarrow \sqrt{\frac{m}{n}}$

Example: $4\sqrt{2}\times 3\sqrt{6}$

Multiply/divide the coefficients and multiply/divide the radicands. Like terms are not necessary.

$$\begin{aligned} 4\sqrt{2}\times 3\sqrt{6} &= (4\times 3)\sqrt{2\times 6} \\ &= 12\sqrt{12} \end{aligned}$$

Examples:

$$5\sqrt{20} \div 3\sqrt{10} = (5 \div 3)\sqrt{20 \div 10}$$

$$= \frac{5}{3}\sqrt{2}$$

$$2\sqrt{3} \times 6\sqrt{8} \div 4\sqrt{12} = 3\sqrt{2}$$

$$3\sqrt{3 \times 8 \div 12}$$

Simplifying Radicals

Many radicals can be simplified.

Example: $\sqrt{\underbrace{27}_{\text{factors}}}$

1. Break down the radicand into two factors - one of which is a perfect square.

$\sqrt{27} = \sqrt{9 \cdot 3}$

2. Apply the multiplication property of radicals to create a coefficient times a radical. $\sqrt{9} \cdot \sqrt{3}$

$\boxed{3\sqrt{3}}$

Example: $3\sqrt{72}$

$3\cdot(\sqrt{36\cdot 2})$

$3(\sqrt{36}\cdot\sqrt{2})$

$3\cdot(6\sqrt{2})$

$18\sqrt{2}$

$9\cdot\underline{\underline{8}}$

OR

$2\cdot 36$

$3(\sqrt{9\cdot 8})$

$3(\sqrt{9}\cdot\sqrt{8})$

$3\cdot(3\sqrt{8}) = 9\sqrt{8}$

$= 9\sqrt{4\cdot 2}$

$= 9\sqrt{4}\cdot\sqrt{2}$

$9\cdot 2\cdot\sqrt{2}$

Example: $4\sqrt{\underline{48}} + 2\sqrt{\overline{12}}$

no match

$4\left(\sqrt{16\cdot 3}\right) + 2\left(\sqrt{4\cdot 3}\right)$

$4\left(\sqrt{16}\cdot\sqrt{3}\right) + 2\left(\sqrt{4}\cdot\sqrt{3}\right)$

$4\cdot\left(4\sqrt{3}\right) + 2\left(2\sqrt{3}\right)$

$\underbrace{16\sqrt{3} + 4\sqrt{3}}_{\text{match}} = 20\sqrt{3}$

Example: $5\sqrt{98}+2\sqrt{200}$

$5\sqrt{49\cdot 2} \quad + \quad 2\sqrt{100\cdot 2}$

$5\sqrt{49}\cdot\sqrt{2} \quad + \quad 2\sqrt{100}\cdot\sqrt{2}$

$5\cdot 7\cdot\sqrt{2} \quad + \quad 2\cdot 10\cdot\sqrt{2}$

$35\sqrt{2} + 20\sqrt{2}$

$55\sqrt{2}$

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Rationalising the Denominator

We do not write expressions with a radical in the denominator.

Example: $\dfrac{6}{\sqrt{3}} \cdot \sqrt{3}$

We get rid of the radical in the denominator by a process called rationalising.

Multiply by a unit fraction that will square the denominator.

$$\frac{6}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{6\sqrt{3}}{\sqrt{3 \cdot 3}} = \frac{6\sqrt{3}}{\sqrt{9}}$$

$$= \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Rationalise the denominators:

1. $\frac{4}{\sqrt{13}} \cdot \left(\frac{\sqrt{13}}{\sqrt{13}}\right) = \frac{4\sqrt{13}}{(\sqrt{13})^2} = \frac{4\sqrt{13}}{13}$

2. $\frac{5\sqrt{3}}{3\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{5\sqrt{6}}{3\sqrt{4}} = \frac{5\sqrt{6}}{3 \cdot 2} = \frac{5\sqrt{6}}{6}$

3. $\dfrac{5\sqrt{6}}{12\sqrt{7}} \cdot \left(\dfrac{\sqrt{7}}{\sqrt{7}}\right) = \dfrac{5\sqrt{42}}{12(\sqrt{7})^2} = \dfrac{5\sqrt{42}}{12 \cdot 7} = \boxed{\dfrac{5\sqrt{42}}{84}}$

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Example: $\dfrac{5}{\sqrt{3}+\sqrt{6}}\left(\dfrac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}}\right) = \dfrac{5\sqrt{3}-5\sqrt{6}}{3-\cancel{\sqrt{18}}+\cancel{\sqrt{18}}-6}$

conjugate

When more than 1 term is in the denominator, we must use conjugates to create a difference of squares.

$$\frac{5\sqrt{3}-5\sqrt{6}}{-3}$$