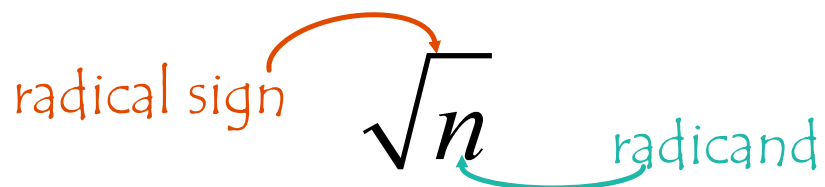


## Radicals

Radicals are expressions that involve a root sign.

$\sqrt{n}$  is called a radical.



## Addition and Subtraction

Adding and subtracting radicals is like algebra - they have to be "like terms" - the radicals must match.

Examples:  $6\sqrt{7} - 4\sqrt{7} = 2\sqrt{7}$

$$5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$$

$$\sqrt{3} + 2\sqrt{5} + 10\sqrt{5} = \text{[orange box]}$$

$$6\sqrt{3} - 2\sqrt{5} = 6\sqrt{3} - 2\sqrt{5}$$

## Multiplication and División

Properties: 1)  $\sqrt{m} \times \sqrt{n} \Leftrightarrow \sqrt{m \times n}$       2)  $\frac{\sqrt{m}}{\sqrt{n}} \Leftrightarrow \sqrt{\frac{m}{n}}$

Example:  $4\sqrt{2} \times 3\sqrt{6}$

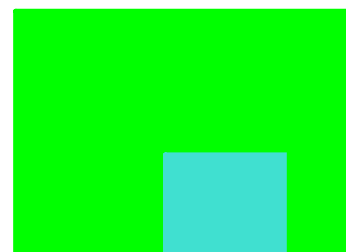


Multiply/divide the coefficients and multiply/divide the radicands. Like terms are not necessary.

$$\begin{aligned} 4\sqrt{2} \times 3\sqrt{6} &= (4 \times 3)\sqrt{2 \times 6} \\ &= 12\sqrt{12} \end{aligned}$$

Examples:

$$\begin{aligned}5\sqrt{20} \div 3\sqrt{10} &= (5 \div 3)\sqrt{20 \div 10} \\ &= \frac{5}{3}\sqrt{2}\end{aligned}$$



$$\begin{aligned}\underline{2}\sqrt{3} \times \underline{6}\sqrt{8} \div \underline{4}\sqrt{12} &= 3\sqrt{2} \\ 3\sqrt{3 \times 8 \div 12} &\quad \uparrow\end{aligned}$$

## Simplifying Radicals

Many radicals can be simplified.

Example:  $\sqrt{27}$  factors

1. Break down the radicand into two factors - one of which is a perfect square.

$$\sqrt{27} = \sqrt{9 \cdot 3}$$

2. Apply the multiplication property of radicals to create a coefficient times a radical.

$$\underline{\underline{3\sqrt{3}}} \quad \underline{\underline{\sqrt{9} \cdot \sqrt{3}}}$$

Example:  $3\sqrt{72}$

$$3 \cdot (\sqrt{36 \cdot 2})$$

$$3(\sqrt{36} \cdot \sqrt{2})$$

$$3(6\sqrt{2})$$

$$18\sqrt{2}$$

or

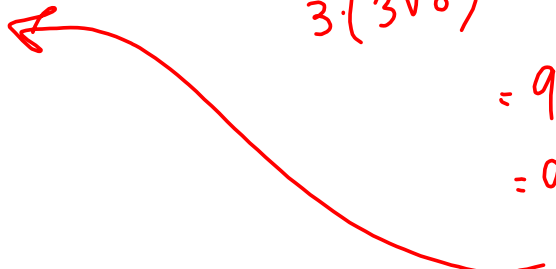
$$3(\sqrt{9 \cdot 8})$$

$$3(\sqrt{9} \cdot \sqrt{8})$$

$$3(3\sqrt{8}) = 9\sqrt{8}$$

$$= 9\sqrt{4 \cdot 2}$$

$$= 9\sqrt{4} \cdot \sqrt{2}$$

$$9 \cdot 2 \cdot \sqrt{2}$$


Example:  $4\sqrt{48} + 2\sqrt{12}$   
no match

$$4(\sqrt{16 \cdot 3}) + 2(\sqrt{4 \cdot 3})$$

$$4(\sqrt{16} \cdot \sqrt{3}) + 2(\sqrt{4} \cdot \sqrt{3})$$

$$4 \cdot (4\sqrt{3}) + 2(2\sqrt{3})$$

$$16\sqrt{3} + 4\sqrt{3}$$

match

$$= 20\sqrt{3}$$

Example:  $5\sqrt{98} + 2\sqrt{200}$

$$5\sqrt{49 \cdot 2} + 2\sqrt{100 \cdot 2}$$

$$5\sqrt{49} \cdot \sqrt{2} + 2\sqrt{100} \cdot \sqrt{2}$$

$$5 \cdot 7 \cdot \sqrt{2} + 2 \cdot 10 \cdot \sqrt{2}$$

$$35\sqrt{2} + 20\sqrt{2}$$

$$55\sqrt{2}$$

Page 15 #4



## Rationalising the Denominator

We do not write expressions with a radical in the denominator.

Example:  $\frac{6}{\sqrt{3}} \cdot \sqrt{3}$

We get rid of the radical in the denominator by a process called rationalising.

Multiply by a unit fraction that will square the denominator.

$$\frac{6}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{6\sqrt{3}}{\sqrt{3 \cdot 3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Rationalise the denominators:

$$1. \quad \frac{4}{\sqrt{13}} \cdot \left(\frac{\sqrt{13}}{\sqrt{13}}\right) = \frac{4\sqrt{13}}{(\sqrt{13})^2} = \frac{4\sqrt{13}}{13}$$

$$2. \quad \frac{5\sqrt{3}}{3\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{5\sqrt{6}}{3\sqrt{4}} = \frac{5\sqrt{6}}{3 \cdot 2} = \frac{5\sqrt{6}}{6}$$

$$3. \frac{5\sqrt{6}}{12\sqrt{7}} \cdot \left(\frac{\sqrt{7}}{\sqrt{7}}\right) = \frac{5\sqrt{42}}{12(\sqrt{7})^2} = \frac{5\sqrt{42}}{12 \cdot 7} = \boxed{\frac{5\sqrt{42}}{84}}$$

Page 17 # 7

Example:  $\frac{5}{\sqrt{3} + \sqrt{6}} \left( \frac{\sqrt{3} - \sqrt{6}}{\sqrt{3} - \sqrt{6}} \right) = \frac{5\sqrt{3} - 5\sqrt{6}}{3 - \cancel{\sqrt{18}} + \cancel{\sqrt{18}} - 6}$

The word "conjugate" is written in red above the fraction  $\frac{\sqrt{3} - \sqrt{6}}{\sqrt{3} - \sqrt{6}}$ . A blue arrow points from the word to the conjugate fraction. Another blue arrow points from the denominator of the resulting fraction to the simplified denominator below.

When more than 1 term is in the denominator, we must use **conjugates** to create a difference of squares.

$$\frac{5\sqrt{3} - 5\sqrt{6}}{-3}$$