

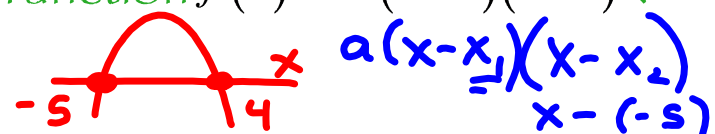
## Second-Degree Function: FACTORED FORM

$$f(x) = a(x - x_1)(x - x_2)$$

$a$  is parameter  $a$  (same as the standard and general forms) - it lets us know if the parabola opens up or down and how wide .

$x_1$  and  $x_2$  are the zeros of the function.

What is the vertex of the function  $f(x) = -3(x-4)(x+5)$ ?

The zeros are  $x_1 = 4$  and  $x_2 = -5$ . 

Since the parabola is symmetrical, the vertex must be exactly halfway between them.

$$h = \frac{x_1 + x_2}{2}$$

The average of the zeros determines the middle.

$$\therefore h = \frac{4 + (-5)}{2} = \frac{-1}{2} \quad \text{and} \quad k = f(h) = -3(-0.5 - 4)(-0.5 + 5)$$

$$k = -3(-4.5)(4.5)$$

$$k = 60.75$$

The vertex of the function is  $V(-0.5, 60.75)$ .

What is the vertex of the function  $g(x) = \frac{1}{2}(x-8)(x-20)$ ?

Zeros:  $\{8, 20\}$

$$h = \frac{8+20}{2} = 14$$

$$k = \frac{1}{2}(14-8)(14-20)$$
$$k = \frac{1}{2}(6)(-6) = -18$$

The vertex is  $V(14, -18)$ .

Convert  $f(x) = -5(x+2)(x-7)$  into...

a) standard form  $\downarrow a = -5$

Need  $h$  &  $k$

$$1) h = \frac{-2+7}{2} = 2.5$$

$$2) k = -5(2.5+2)(2.5-7)$$

$$k = -5(4.5)(-4.5)$$

$$k = 101.25$$

$$\therefore f(x) = -5(x-2.5)^2 + 101.25$$

b) general form

$$f(x) = -5(x+2)(x-7)$$

FOIL

$$f(x) = -5(x^2 - 5x - 14)$$

$$\therefore f(x) = -5x^2 + 25x + 70$$

Convert  $f(x) = -(x-7)^2 + 9$  into factored form.

$$a = -1$$

Find the zeros (let  $y = 0$ ).

$$0 = -(x-7)^2 + 9$$

$$\frac{-9}{-1} = \frac{-(x-7)^2}{-1}$$

$$9 = (x-7)^2$$

$$\pm\sqrt{9} = x-7$$

$$\pm 3 = x-7 \longrightarrow \begin{array}{l} 1) \quad 3 = x-7 \\ 2) \quad -3 = x-7 \end{array}$$

$$10 = x_1 \quad 4 = x_2$$

$$f(x) = -(x-10)(x-4)$$

or

$$x = h \pm \sqrt{\frac{-k}{a}}$$

$$x = 7 \pm \sqrt{\frac{-9}{-1}}$$

$$= 7 \pm \sqrt{9}$$

$$= 7 \pm 3$$

Convert  $y = 3x^2 - 35x - 12$  into factored form.

$a = 3$  Find the zeros (let  $y = 0$ ).

$$0 = 3x^2 - 35x - 12$$

1) By factoring:  $m+n = -35$  }  $-36$  or  $m \times n = -36$

$$0 = 3x^2 + x - 36x - 12$$

$$0 = x(3x + 1) - 12(3x + 1)$$

$$0 = (3x + 1)(x - 12)$$

$$3x + 1 = 0$$

$$3x = -1 \rightarrow x_1 = -\frac{1}{3}$$

$$x - 12 = 0 \rightarrow x_2 = 12$$

2) Quadratic Equation

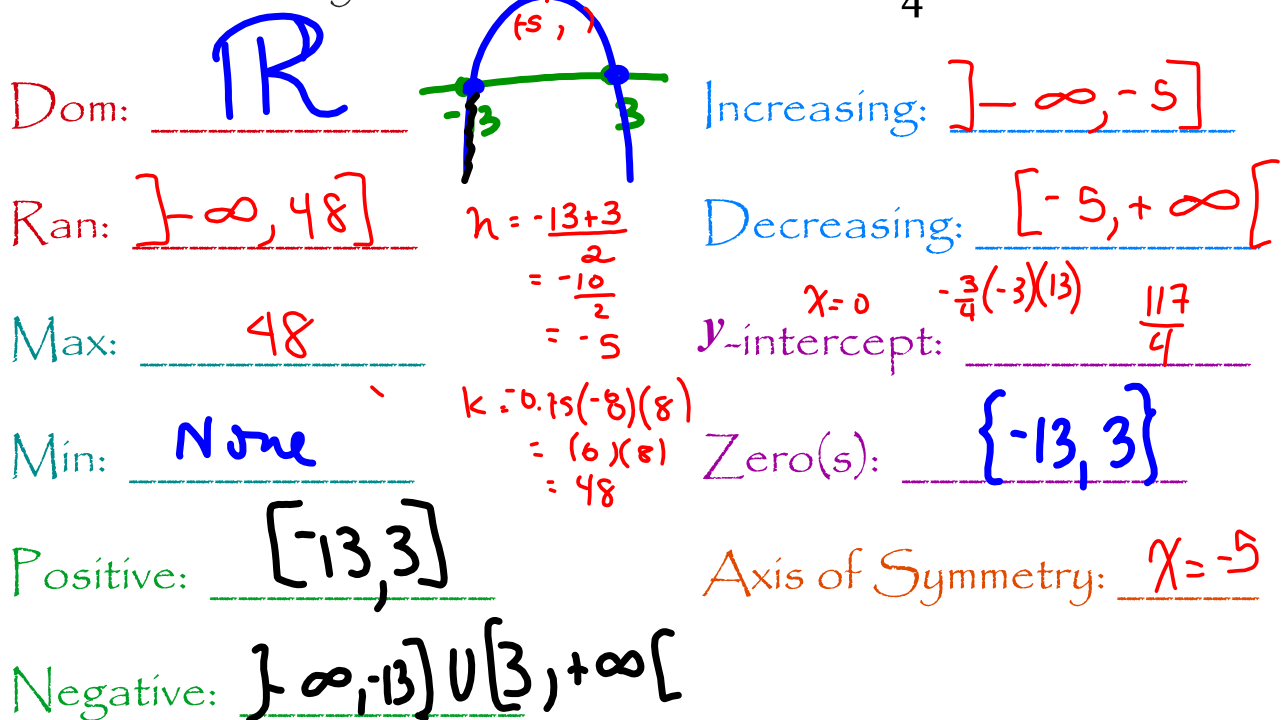
$$x = \frac{-(-35) \pm \sqrt{1225 - 4(3)(-12)}}{6}$$

$$x = \frac{35 \pm \sqrt{1369}}{6} = \frac{35 \pm 37}{6}$$

$$x_1 = \frac{35-37}{6} \quad x_2 = \frac{35+37}{6}$$

$$y = 3 \left( x + \frac{1}{3} \right) (x - 12)$$

Provide a study of the function  $f(x) = -\frac{3}{4}(x-3)(x+13)$ .



Provide a study of the function  $f(x) = 2x(x+7)$ .

Dom:  $\mathbb{R}$    $= 2(x-0)(x+7)$  Increasing:  $[-3.5, +\infty[$

Ran:  $[-24.5, +\infty[$   $h = -3.5$  Decreasing:  $] -\infty, -3.5]$

Max: None  $k = 2(-3.5)(3.5)$   $y$ -intercept: 0  
 $= -7 \cdot 3.5$

Min: -24.5  $= -24.5$  Zero(s):  $\{-7, 0\}$

Positive:  $] -\infty, -7] \cup [0, +\infty[$  Axis of Symmetry:  $x = -3.5$

Negative:  $[7, 0]$