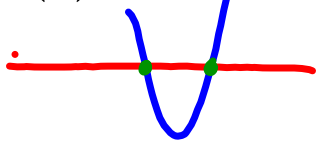
Over what interval(s) is the function $f(x) = 8x^2 + 6x - 2$ positive?



1) Find the zeros $0 = 8x^2 + 6x - 2$

$0 = 4x^2 + 3x - 1$

$0 = 4x^2 + 4x - x - 1$

$0 = 4x(x+1) - 1(x+1)$

$0 = (x+1)(4x-1)$

$x+1 = 0 \quad 4x-1 = 0$

$x = -1 \quad x = 1/4$

$x = \frac{-6 \pm \sqrt{36 - 4(8)(-2)}}{16}$

$x = \frac{-6 \pm \sqrt{100}}{16}$

$x = \frac{-6 \pm 10}{16}$

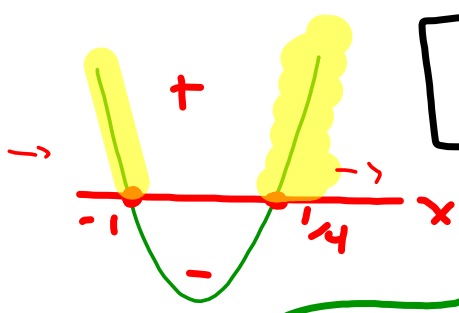
$x_1 = \frac{4}{16} = \frac{1}{4}$ $\quad x_2 = \frac{-16}{16} = -1$

The zeros are -1 and $\frac{1}{4}$.

2a) Use a sketch

$8 = a$

zeros $-1, \frac{1}{4}$



$]-\infty, -1] \cup [\frac{1}{4}, +\infty[$

2b) Use a number line

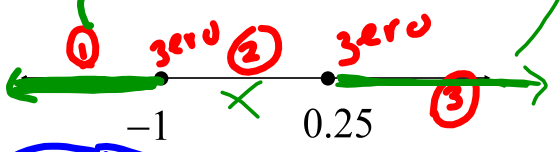
① test $x = -2$

$y = 8x^2 + 6x - 2$

$= 8(-2)^2 + 6(-2) - 2$

$= 32 - 12 - 2$

$= 18$ ⊕



② test $x = 0$

$y = 0 + 0 - 2$

$y = -2$ ⊖

③ test $x = 1$

$y = 8 + 6 - 2 = 12$ ⊕

Convert $y = -2(x-5)^2 - 10$ to general form.

$y = -2(x^2 - 10x + 25) - 10$

$y = -2x^2 + 20x - 50 - 10$

$y = -2x^2 + 20x - 60$

Converting from General Form to Standard Form

Completing the Square

a) Form: $x^2 + bx + c$ $a = 1$

Recall the perfect square trinomial

$$(x+n)^2 = x^2 + 2nx + n^2$$

Example: $x^2 - 10x - 24$

Step 1: Divide the coefficient of x (b) by 2.

$$-10 \div 2 = -5$$

Step 2: Square this value, then simultaneously add AND subtract the result within the polynomial.

$$(-5)^2 = 25$$

P.S.T

$$x^2 - 10x + 25 - 25 - 24$$

Step 3: Factor the perfect square trinomial part and add the two constants .

$$1x^2 - 10x + 25 - 25 - 24$$

$$(x-5)^2 - 49$$

Complete the square:

a) $x^2 + 8x - 20$

① $8 \div 2 = 4$

② $4^2 = 16$ - add/subtract

$x^2 + 8x + 16 - 16 - 20$

$(x + 4)^2 - 36$

b) $x^2 - 5x - 14$

① $-5 \div 2 = -2.5$

② $(-2.5)^2 = 6.25$ add/sub

$x^2 - 5x + 6.25 - 6.25 - 14$

$(x - 2.5)^2 - 20.25$

Example: Convert the function $f(x) = x^2 - 16x + 47$ to standard form.

$f(x) = x^2 - 16x + 47$

① $-16 \div 2 = -8$

② $(-8)^2 = 64$ add/sub

$f(x) = x^2 - 16x + 64 - 64 + 47$

$f(x) = (x-8)^2 - 17$

Example: Convert the function $y = x^2 + 9x - 2$ to standard form.

① $9 \div 2 = 4.5$
② $4.5^2 = 20.25$

$y = x^2 + 9x - 2$

$y = x^2 + 9x + 20.25 - 20.25 - 2$

$y = (x + 4.5)^2 - 22.25$

b) Form ax^2+bx+c $a \neq 1$

Example: $9x^2-18x-7$

Step 1: Factor a out of only the first two terms .

$$9\left(x^2-2x\right)-7$$

Step 2: Inside the parentheses , follow the steps to complete the square .

$$9\left(x^2-2x\right)-7$$

$-2\div 2=-1$

$(-1)^2=1$ add/sub

$$9\left(x^2-2x+1-1\right)-7$$

Step 3a: Factor the perfect square trinomial.

$$9\left(x^2-2x+1-1\right)-7=9\left((x-1)^2-1\right)-7$$

Step 3b: Multiply both terms by the a factored out in step 1.

$$9\left((x-1)^2-1\right)-7=9(x-1)^2-9-7$$

Add the constants.

$$9(x-1)^2-16$$

Complete the square.

a) $3x^2+12x-4$

$3(x^2+4x)-4$

$4 \div 2 = 2$

$2^2 = 4$ add/sub

$3(x^2+4x+4-4)-4$

$3((x+2)^2-4)-4$

$3(x+2)^2-12-4$

$3(x+2)^2-16$

b) $4x^2-2x+7$

$4(x^2-\frac{1}{2}x)+7$

$-\frac{1}{2} \div 2 = -\frac{1}{4}$

$(-\frac{1}{4})^2 = \frac{1}{16}$

$4(x^2-\frac{1}{2}+\frac{1}{16}-\frac{1}{16})+7$

$4((x-\frac{1}{4})^2-\frac{1}{16})+7$

$4(x-\frac{1}{4})^2-\frac{1}{4}+7$

$4(x-0.25)^2+6.75$

Example: Convert the function $f(x)=0.5x^2-3x-4$ to standard form.

Convert to standard form by completing the square .

$$f(x)=0.5x^2-3x-4$$

$$f(x)=0.5\left(x^2-6x\right)-4$$

$-6\div 2=-3$

$(-3)^2=9$

$$f(x)=0.5\left(x^2-6x+9-9\right)-4$$

$$f(x)=0.5\left((x-3)^2-9\right)-4$$

$$f(x)=0.5(x-3)^2-4.5-4$$

$-(0.5\times 9)$

$$f(x)=0.5(x-3)^2-8.5$$

Example: Determine the vertex of the function

$$f(x) = -2x^2 - 6x + 7$$

$$f(x) = -2x^2 - 6x + 7$$

$3 \div 2 = 1.5$

$1.5^2 = 2.25$

$$f(x) = -2\left(x^2 + 3x\right) + 7$$

$$f(x) = -2\left(x^2 + 3x + 2.25 - 2.25\right) + 7$$

$$f(x) = -2\left((x + 1.5)^2 - 2.25\right) + 7$$

$$f(x) = -2(x + 1.5)^2 + 4.5 + 7$$

$$f(x) = -2(x + 1.5)^2 + 11.5$$ standard form

$$\therefore V(-1.5, 11.5)$$