1.

$$\frac{3n+20}{n-4} = \frac{3n^2 + 16n + 5}{n^2 - 25}$$

$$\frac{3n+20}{n-4} = \frac{(3n+1)(n+5)}{(n+5)(n-5)}$$

$$n \neq \{-5, 4, 5\}$$

$$\frac{3n+20}{n-4} = \frac{3n+1}{n-5}$$

$$(3n+20)(n-5) = (n-4)(3n+1)$$
$$3n^2 - 15n + 20n - 100 = 3n^2 + n - 12n - 4$$
$$3n^2 + 5n - 100 = 3n^2 - 11n - 4$$
$$5n - 100 = -11n - 4$$
$$16n - 100 = -4$$
$$16n = 96$$
$$\boxed{n = 6}$$

(right margin work)

$m \times n = 3 \cdot 5 = 15 \quad \rangle \; 1, 15$
$m + n = 16$

$3n^2 + n + 15n + 5$
$n(3n+1) + 5(3n+1)$
$(3n+1)(n+5)$

2.

$$\frac{4x^2 - 49}{2x^2 - 5x - 7} \div \frac{1}{x+1}$$

$$\frac{(2x+7)(2x-7)}{(x+1)(2x-7)} \div \frac{1}{x+1}$$

$$x \neq \{-1, \tfrac{7}{2}\} \quad \text{(not necessary)}$$

$$\frac{(2x+7)(2x-7)}{(x+1)(2x-7)} \times \frac{(x+1)}{1}$$

$$\frac{(2x+7)(2x-7)(x+1)}{(x+1)(2x-7)}$$

$$\boxed{2x - 7}$$

(right margin work)

$2x^2 - 5x - 7$

$m \times n = -14 \quad \rangle \; -7, 2$
$m + n = -5$

$2x^2 + 2x - 7x - 7$
$2x(x+1) - 7(x+1)$
$(x+1)(2x-7)$

3. $\dfrac{(2x-5)(3x-5)}{9x^2-25} + \dfrac{6x^2+5x}{3x+5}$

$\dfrac{(2x-5)(3x-5)}{(3x+5)(3x-5)} + \dfrac{x(6x+5)}{3x+5}$

$\dfrac{2x-5}{3x+5} + \dfrac{6x^2+5x}{3x+5}$

$\dfrac{6x^2+7x-5}{3x+5}$ $\longrightarrow$ $6x^2+7x-5 \Rightarrow$ $\begin{array}{l} m\times n = -30 \\ m+n = 7 \end{array}\Big)\, 10, -3$

$6x^2-3x+10x-5$
$3x(2x-1) +5(2x-1)$
$(2x-1)(3x+5)$

$\dfrac{(2x-1)(3x+5)}{(3x+5)}$

$\boxed{2x-1}$

4. $\dfrac{4x^2-25}{x+4} \times \dfrac{x^2+5x+4}{2x^2-3x-5}$ $\longrightarrow$ $\begin{array}{l} m\times n = -10 \\ m+n = -3 \end{array}\Big)\, -5, 2$

$\dfrac{(2x+5)(2x-5)}{x+4} \times \dfrac{(x+4)(x+1)}{(x+1)(2x-5)}$

$2x^2+2x-5x-5$
$2x(x+1) -5(x+1)$
$(x+1)(2x-5)$

$\dfrac{(2x+5)(2x-5)(x+4)(x+1)}{(x+4)(x+1)(2x-5)}$

$\boxed{2x+5}$

5. $\dfrac{x+4}{x^2-49} + \dfrac{3}{x-7}$

$\dfrac{x+4}{(x+7)(x-7)} + \dfrac{3}{x-7}$

$\dfrac{x+4}{(x+7)(x-7)} + \dfrac{3}{x-7}\left(\dfrac{x+7}{x+7}\right)$

$\dfrac{x+4}{(x+7)(x-7)} + \dfrac{3x+21}{(x+7)(x-7)}$

$\dfrac{4x+25}{(x+7)(x-7)}$ or $\dfrac{4x+25}{x^2-49}$

6. $x+2 - \dfrac{x^2+4x+3}{x^2-9}$

$x+2 - \left(\dfrac{(x+3)(x+1)}{(x+3)(x-3)}\right)$

$x+2 - \dfrac{x+1}{x-3}$

$\left(\dfrac{x-3}{x-3}\right)\dfrac{x+2}{1} - \dfrac{x+1}{x-3}$

$\dfrac{x^2-x-6}{x-3} - \dfrac{(x-1)}{x-3}$

$\dfrac{x^2-2x-5}{x-3}$

7.
$$\text{Area rectangle} = (2x+5)(x) = 2x^2 + 5x$$
$$\text{Area square} = (x+10)^2 = x^2 + 20x + 100$$
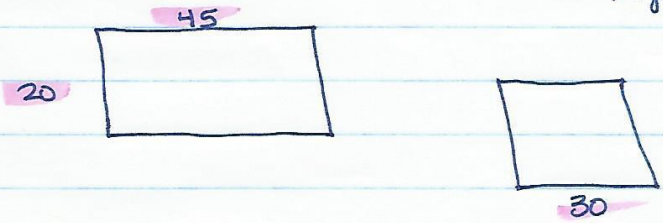
$$2x^2 + 5x = x^2 + 20x + 100$$
$$x^2 - 15x - 100 = 0$$
$$(x-20)(x+5) = 0$$

$$x - 20 = 0 \qquad\qquad x + 5 = 0$$
$$x = 20 \qquad\qquad x = -5$$
$$\text{Reject}$$

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1)
$$\frac{n-5}{n^2 + 4n - 12} = \frac{1}{n^2 + 4n - 12} - \frac{1}{3}$$

$$\frac{n-5}{(n+6)(n-2)} = \frac{1}{(n+6)(n-2)} - \frac{1}{3} \qquad\Rightarrow n \neq \{-6, 2\}$$

$$\underbrace{}_{\text{same denominator}}$$

$$\frac{n-5}{(n+6)(n-2)} - \frac{1}{(n+6)(n-2)} = -\frac{1}{3}$$

$$\frac{n-6}{(n+6)(n-2)} = -\frac{1}{3}$$

$$\frac{n-6}{n^2 + 4n - 12} = \frac{-1}{3}$$

$$3n - 18 = -n^2 - 4n + 12$$
$$n^2 + 7n - 30 = 0$$
$$(n+10)(n-3) = 0$$
$$n + 10 = 0 \quad \text{or} \quad n - 3 = 0$$

$$n = \{-10, 3\}$$

3) $\dfrac{1}{n^2+9n+20} = \dfrac{1}{n+5} - \dfrac{3n^2-15n-72}{n^2+9n+20}$

↑ ↑ same denominator

$\dfrac{1}{n^2+9n+20} + \dfrac{3n^2-15n-72}{n^2+9n+20} = \dfrac{1}{n+5}$

$3(n^2-5n-24)$
$3(n-8)(n+3)$

$\dfrac{1}{(n+5)(n+4)} + \dfrac{(n-8)(n+3)}{(n+5)(n+4)} = \dfrac{1}{n+5}$

$\Rightarrow n \neq \{-5, -4\}$

$\dfrac{1}{(n+5)(n+4)} + \dfrac{3n^2-15n-72}{(n+5)(n+4)} = \dfrac{1}{n+5}$

$\dfrac{3n^2-15n-71}{(n+5)(n+4)} = \dfrac{1}{(n+5)}$

cross multiplying would give a 3rd degree equation, so, instead, make common denominators

$\dfrac{3n^2-15n-71}{(n+5)(n+4)} = \dfrac{1}{n+5}\left(\dfrac{n+4}{n+4}\right)$

$\dfrac{3n^2-15n-71}{(n+5)(n+4)} = \dfrac{n+4}{(n+5)(n+4)}$

same denominators ∴ same numerators

$\therefore \quad 3n^2-15n-71 = n+4$

$3n^2-16n-75 = 0$

$\begin{array}{c} -25 \\ 9 \end{array} \left\{ \begin{array}{l} m \times n = -225 \\ m+n = -16 \end{array} \right.$

$3n^2+9n-25n-75 = 0$

$3n(n+3) - 25(n+3) = 0$

$(n+3)(3n-25) = 0$

$n+3=0 \qquad 3n-25=0 \qquad \Rightarrow \quad 3n=25$

$n = \left\{-3, \dfrac{25}{3}\right\}$

$n = \dfrac{25}{3}$

6) $\dfrac{7}{n^2} = \dfrac{2n+14}{n^3-8n^2} - \dfrac{n^2+n-2}{n^3-8n^2}$

$\dfrac{7}{n^2} = \dfrac{2(n+7)}{n^2(n-8)} - \dfrac{(n+2)(n-1)}{n^2(n-8)}$, $n \neq \{0, 8\}$

$\dfrac{7}{n^2} = \dfrac{2n+14 - (n^2+n-2)}{n^2(n-8)}$

$\dfrac{7}{n^2} = \dfrac{-n^2 + n + 16}{n^2(n-8)}$ exponents are too large — cross multiplying won't give a second-degree equation so make common denominators

$\left(\dfrac{n-8}{n-8}\right)\dfrac{7}{n^2} = \dfrac{-n^2+n+16}{n^2(n-8)}$

$\dfrac{7n-56}{n^2(n-8)} = \dfrac{-n^2+n+16}{n^2(n-8)}$

$\therefore \quad 7n - 56 = -n^2 + n + 16$

$n^2 + 6n - 72 = 0$

$(n+12)(n-6) = 0$

$n + 12 = 0 \quad$ or $\quad n - 6 = 0$

$\boxed{n = \{-12, 6\}}$

12). $\dfrac{x-6}{x} = \dfrac{21x+28}{x} - \dfrac{x+8}{x^2+2x}$

$\dfrac{x-6}{x} = \dfrac{7(3x+4)}{x} - \dfrac{x+8}{x(x+2)}$ make common denominators

$\left(\dfrac{x+2}{x+2}\right)\left(\dfrac{x-6}{x}\right) = \dfrac{(21x+28)}{x}\left(\dfrac{x+2}{x+2}\right) - \dfrac{x+8}{x(x+2)}$, $x \neq \{-2, 0\}$

$\dfrac{x^2-4x-12}{(x+2)(x)} = \dfrac{21x^2+70x+56}{x(x+2)} - \dfrac{(x+8)}{x(x+2)}$

$\dfrac{x^2-4x-12}{x(x+2)} = \dfrac{21x^2+69x+48}{x(x+2)}$

$\therefore \quad x^2 - 4x - 12 = 21x^2 + 69x + 48$

$0 = 20x^2 + 73x + 60$

12)
$$x = \frac{-73 \pm \sqrt{73^2 - 4(20)(60)}}{2(20)} = \frac{-73 \pm \sqrt{529}}{40}$$

$$= \frac{-73 \pm 23}{40}$$

$$x_1 = \frac{-73 - 23}{40} \qquad\qquad x_2 = \frac{-73 + 23}{40}$$

$$x_1 = -\frac{96}{40} = \frac{-12}{5} \qquad\qquad x_2 = -\frac{50}{40} = -\frac{5}{4}$$

$$x = \{-2.4, 1.25\}$$

13)
$$\frac{1}{3x^2 - 21x + 36} = \frac{1}{(3x - 9)} - \frac{x^2 - 2x - 15}{3x^2 - 21x + 36}$$

$$\frac{1}{3(x^2 - 7x + 12)} = \frac{1}{3(x - 3)} - \frac{(x - 5)(x + 3)}{3(x^2 - 7x + 12)}$$

$$\frac{1}{3(x - 4)(x - 3)} = \frac{1}{3(x - 3)} - \frac{(x - 5)(x + 3)}{3(x - 4)(x - 3)} \qquad , \quad x \neq \{3, 4\}$$

$$\frac{1}{3(x - 4)(x - 3)} = \left(\frac{x - 4}{x - 4}\right)\left(\frac{1}{3(x - 3)}\right) - \frac{(x - 5)(x + 3)}{3(x - 4)(x - 3)}$$

$$\frac{1}{3(x - 4)(x - 3)} = \frac{x - 4}{3(x - 4)(x - 3)} - \frac{(x^2 - 2x - 15)}{3(x - 4)(x - 3)}$$

$$\frac{1}{3(x - 4)(x - 3)} = \frac{-x^2 + 3x + 11}{3(x - 4)(x - 3)}$$

$$\therefore \quad 1 = -x^2 + 3x + 11$$

$$x^2 - 3x - 10 = 0$$

$$(x - 5)(x + 2) = 0$$

$$x - 5 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = \{-2, 5\}$$

27.

$$\frac{x-5}{2x^2-4x} + \frac{x^2-6x-7}{2x^2-4x} = \frac{1}{x^2-2x}$$

$$\frac{x-5}{2x(x-2)} + \frac{x^2-6x-7}{2x(x-2)} = \frac{1}{x(x-2)} \qquad \Rightarrow \quad x \neq \{0, 2\}$$

$$\frac{x^2-5x-12}{2x(x-2)} = \frac{1}{x(x-2)}\left(\frac{2}{2}\right)$$

$$\therefore x^2 - 5x - 12 = 2$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x-7=0 \quad \text{or} \quad x+2=0$$

$$x = \{-2, 7\}$$

28.

$$\frac{n^2-25}{n^2-11n+28} + \frac{n^2+10n+16}{n^2-11n+28} = \frac{1}{n-7}$$

$$\frac{n^2-25}{(n-7)(n-4)} + \frac{n^2+10n+16}{(n-7)(n-4)} = \frac{1}{(n-7)} \qquad \Rightarrow \quad n \neq \{4, 7\}$$

$$\frac{2n^2+10n-9}{(n-7)(n-4)} = \frac{1}{n-7} \qquad \text{cross multiply}$$

$$(n-7)(2n^2+10n-9) = (n-7)(n-4)$$

Divide both sides by $(n-7)$ instead of multiplying — would give a 3rd degree

$$\frac{(n-7)(2n^2+10n-9)}{(n-7)} = \frac{(n-7)(n-4)}{(n-7)}$$

$$2n^2+10n-9 = n-4$$

$$2n^2 + 9n - 5 = 0 \qquad m \cdot n = -10 \Big\} \ 10, -1$$

$$2n^2 - n + 10n - 5 = 0 \qquad m+n = 9$$

$$n(2n-1) + 5(2n-1) = 0$$

$$(2n-1)(n+5) = 0$$

$$2n-1=0 \quad \text{or} \quad n+5=0$$

$$2n = 1$$

$$n = \frac{1}{2} \qquad\qquad n = \{-5, \tfrac{1}{2}\}$$

52) $\dfrac{1}{n} + \dfrac{8n^2 - 55n - 7}{4n^2 + 8n} = \dfrac{n-7}{n}$

<u>same denominator</u>

$\dfrac{8n^2 - 55n - 7}{4n(n+2)} = \dfrac{n-7}{n} - \dfrac{1}{n}$, $n \ne \{-2, 0\}$

$\dfrac{\cdot (n-7)(8n+1)}{4n(n+2)} = \dfrac{n-8}{n}$

cross multiply

$8n^2 - 55 - 7$ $m \times n = ^-56$
$8n^2 - 56n + n - 7$ $m + n = -56$
$8n(n-7) + 1(n-7)$

$\underline{n}(n-7)(8n+1) = 4\underline{n}(n+2)(n-8)$

divide both sides by n (avoids a 3ʳᵈ degree equation)

$(n-7)(8n+1) = 4(n+2)(n-8)$

$8n^2 - 55n - 7 = 4(n^2 - 6n - 16)$

$8n^2 - 55n - 7 = 4n^2 - 24n - 64$

$4n^2 - 31n + 57 = 0$ $m \times n = 228$ ⟩ $-12, -19$
$4n^2 - 12n - 19n + 57 = 0$ $m + n = -31$
$4n(n-3) - 19(n-3) = 0$
$(n-3)(4n-19) = 0$

$n - 3 = 0$ or $4n - 19 = 0$ ⟹ $4n = 19$

$n = \left\{ 3, \dfrac{19}{4} \right\}$ $n = \dfrac{19}{4}$