# Solving Second-Degree Equations

Zero Product Principle - a product of factors is equal to zero if and only if at least one of the factors is equal to zero.

Example: if $5 \times \blacksquare = 0$ $\quad \blacksquare = 0$

We will use this and factoring techniques to solve equations.

Example: Solve $\boxed{x^2 - 10x = 0}$

*factor the LHS* $\quad x\underbrace{(x-10)}_{F_2} = 0$

$F_1 \cdot F_2 = 0$

$\therefore$ *either* $x$ *or* $x-10$ *must be 0.*

*So* $x = 0$ *or* $x - 10 = 0$

$x = 10$

$$\boxed{x = \{0, 10\}}$$

Example: Solve $\boxed{x^2 \underbrace{-2x}_{m+n} \underbrace{-15}_{m \times n} = 0}$

*factor the LHS*

$(x-5)(x+3) = 0$

$F_1 \cdot F_2 = 0$

$x-5=0$ or $x+3=0$

$x=5$ or $x=-3$

$x=\{-3,5\}$

Example: Solve $\underbrace{-2x^2 - 5x + 3}_{}= 0$

$m \times n = -6$

$m + n$

Factor the *LHS*

$-2x^2 - 6x + 1x + 3 = 0$

$-2x(x+3) + 1(x+3) = 0$

$(x+3)(-2x+1) = 0$

$x + 3 = 0$ or $-2x + 1 = 0$

$\boxed{x = -3}$ $-2x = -1$

$\boxed{x = 0.5 \text{ or } \frac{1}{2}}$

Solve: $\boxed{2x^2 - x = 6}$

$\quad\quad\quad\quad -6 \quad -6$

*Make the equation equal to* 0*, then factor the LHS.*

$-12 = m \times n$

$\underbrace{-1 = m + n}_{-4,\ +3}$

$2x^2 - x - 6 = 0$

$2x^2 - 4x + 3x - 6 = 0$

$2x(x-2) + 3(x-2) = 0$

$(x-2)(2x+3) = 0$

$x - 2 = 0$ OR $2x + 3 = 0$

$x = 2$ $\quad\quad 2x = -3$

$x = -\frac{3}{2}$

Solve $\boxed{4x^2 - 36 = 0}$

$(2x-6)(2x+6) = 0$

$2x-6=0$
$2x=6$
$x=3$

$2x+6=0$
$2x=-6$
$x=-3$

$4x^2 - 36 = 0$
$+36 \quad +36$

$4x^2 = 36$

$\frac{4x^2}{4} = \frac{36}{4}$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$\Rightarrow x = \pm 3$

Solve $2x^2 - 50 = 0$

$2x^2 = 50$

$x^2 = 25$

$x = \sqrt{25} = \pm 5$

$2(x^2 - 25) = 0$

$2(x+5)(x-5) = 0$

$x + 5 = 0$

$x = -5$

or $x - 5 = 0$

$x = 5$

Solve $5x^2 - 35 = 0$

$$5x^2 = 35$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$

$$\boxed{\therefore x = \left\{-\sqrt{7}, \sqrt{7}\right\}}$$

Example: Solve $14x^2 + 28 = 0$

$$14(x^2+2)=0$$
$$x^2+2=0$$
$$x^2=-2 \rightarrow x=\sqrt{-2}$$

You can't calculate the square root of a negative number.

<u>No Real solution.</u>

Example:

Solve $10x^2-4x-7=4\cancel{x}^2-\cancel{1}1x+13$

$-4x^2 \quad +11x \quad -13 \quad -4x^2 \quad +11x \quad -13$

$6x^2+7x-20=0$

$m \times n = -120$
$\underbrace{m+n = 7}_{-8,\ +15}$

$6x^2-8x+15x-20=0$

$2x(3x-4)+5(3x-4)=0$

$(3x-4)(2x+5)=0$

$3x-4=0$ or $2x+5=0$

$3x=4 \qquad 2x=-5$

$x=4/3 \qquad x=\frac{-5}{2}$

Example: The length of a rectangle is $5cm$ longer than its width. If the area is equal to $150cm^2$ **what is the numerical value of the perimeter of the rectangle?**

width: $x$

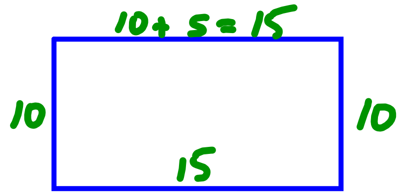
length: $x+5$

Area: $x(x+5)=150$

$$x^2+5x=150$$

$$x^2+5x-150=0$$

$$(x+15)(x-10)=0$$




$x+15=0$ or $x-10=0$

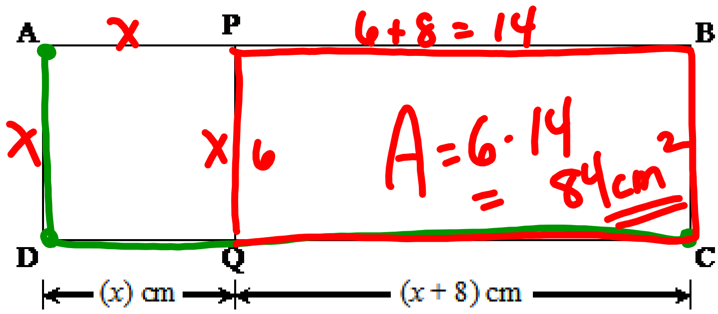
$\boxed{x=-15}$ or $\boxed{x=10}$

Does not make sense

Perimeter = 50 cm

Example: In the figure, $\overline{PQ}$ divides rectangle $ABCD$ into two quadrilaterals: square $APQD$ and rectangle $PBCQ$. The area of rectangle $ABCD$ is $120cm^2$. In addition, $m\overline{DQ} = (x)cm$ and $m\overline{QC} = (x+8)cm$.

What is the numerical area of rectangle PBCQ?




$x(2x+8)=120$

$2x^2+8x=120$

$$\frac{2x^2+8x-120}{2}=\frac{0}{2}$$

$x^2+4x-60=0$

$(x+10)(x-6)=0$

$x+10=0$ or $x-6=0$

$x=\{-10,6\}$

Reject

$x+x+8=2x+8$

$\therefore \boxed{x=6}$