Today, a mother's age is two years more than double her son's age. In ten years, the product of their ages will be 2040. How old are they today ?

2 variables
2 times

	Today	Future
Mother's age	$2x+2$	$2x+12$
Son's age	x	$x+10$

$(2x+12)(x+10)=2040$

$2x^2+20x+12x+120=2040$

$2x^2+32x+120=2040$

$2x^2+32x-1920=0$

$\div 2$

$2(x^2+16x-960)=0 \div 2$

$x^2+16x-960=0$

mxn = -960
m+n = 16
40
-24

$(x+40)(x-24)=0$

$x+40=0$ or $x-24=0$

$x=-40$ Does not make sense

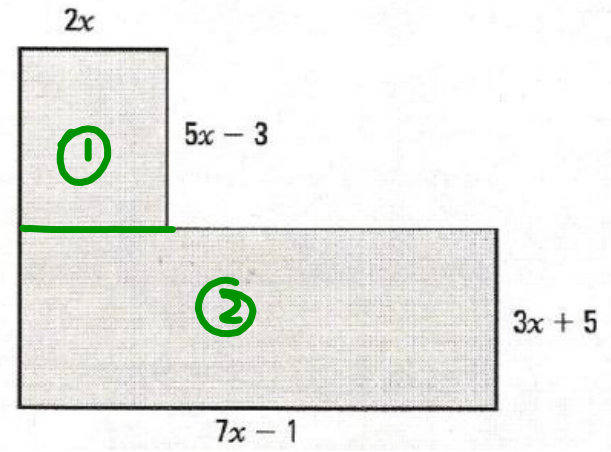
$x=24$

$\therefore x=24 \Rightarrow$ son

mother $= 2(24)+2$
$= 48+2$
$= 50$ yr's old

The Quadratic Formula

The area of this figure is equal to $103.75cm^2$.
Determine the numerical length of each side .



$$2x(5x-3)+(3x+5)(7x-1)=103.75$$

$$10x^2-6x+21x^2-3x+35x-5=103.75$$

$$31x^2+26x-5=103.75$$

$$-103.75 \quad -103.75$$

$$31x^2+26x-108.75=0$$

$$31x^2 + 26x - 108.75 = 0$$

$$m \times n = -3371.25$$
$$m + n = 26$$

The quadratic formula provides a solution to any quadratic (second-degree) equation.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $ax^2 + bx + c = 0$

$$\underset{a}{31}x^2 + \underset{b}{26}x \underset{c}{- 108.75} = 0$$

$$x = \frac{-26 \pm \sqrt{26^2 - 4(31)(-108.75)}}{2(31)}$$

$= \sqrt{14\,161}$

$= 62$

$$x = \frac{-26 \pm \sqrt{14161}}{62} \quad = 119$$

1 $$x = \frac{-26 + 119}{62}$$

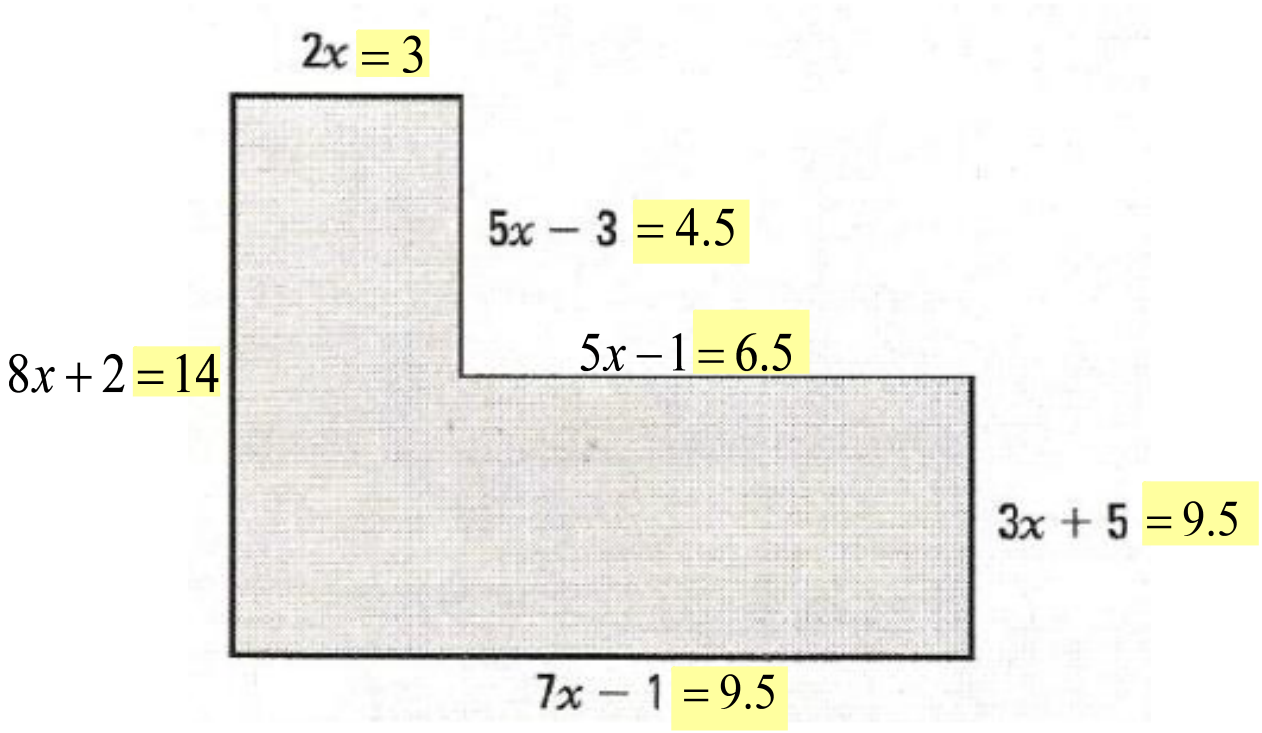
$$x = \frac{93}{62} = 1.5$$

2 $$x = \frac{-26 - 119}{62}$$

$$x = \frac{-145}{62} \approx -2.34$$

Does not work

$$\therefore x = 1.5$$


$2x = 3$
$5x - 3 = 4.5$
$5x - 1 = 6.5$
$8x + 2 = 14$
$3x + 5 = 9.5$
$7x - 1 = 9.5$

Example:

Solve $15x^2 - 2x - 8 = 0$ (a = 15, b = −2, c = −8)

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

answer is (+)

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(15)(-8)}}{2(15)} \Rightarrow \sqrt{4 - 4(15)(-8)}$$

$$x = \frac{2 \pm \sqrt{4 + 480}}{30}$$

$$x = \frac{2 \pm \sqrt{484}}{30}$$

$$x = \frac{2 \pm 22}{30}$$

$x_1 = \frac{2 + 22}{30} = \frac{24}{30} = \frac{4}{5}$ or $x_2 = \frac{2 - 22}{30} = \frac{-20}{30} = -\frac{2}{3}$

Solve $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

a) $1x^2 - 6x - 91 = 0$

$x = \dfrac{+6 \pm \sqrt{(-6)^2-4(1)(-91)}}{2(1)}$

$x = \dfrac{6 \pm \sqrt{400}}{2}$

$x = \dfrac{6 \pm 20}{2}$

$x_1 = \dfrac{6+20}{2}$ or $x_2 = \dfrac{6-20}{2}$

$= \dfrac{26}{2}$ $= -13$ $= -\dfrac{14}{2}$ $= -7$

b) $9x^2 + 30x + 25 = 0$

$x = \dfrac{-30 \pm \sqrt{900-4(9)(25)}}{2(9)}$

$x = \dfrac{-30 \pm \sqrt{0}}{18}$

$x = \dfrac{-30 \pm 0}{18} = \dfrac{-30}{18}$

$x = -\dfrac{5}{3}$

c) $5x^2 + 9x + 12 = 0$

$$x = \frac{-9 \pm \sqrt{81 - 4(5)(12)}}{2(5)}$$

$$x = \frac{-9 \pm \sqrt{-159}}{10}$$

No real value

No solution

The Discriminant (Δ)

- the portion under the root sign: $\Delta = b^2 - 4ac$

If ...

$b^2 - 4ac > 0$ (+) there are 2 real answers

$b^2 - 4ac = 0$ there is one real answer

$b^2 - 4ac < 0$ (-) there are no real answers