# Solving Second-Degree Equations

Zero Product Principle - a product of factors is equal to zero if and only if at least one of the factors is equal to zero.

Example: if $5 \times \blacksquare = 0$ $\blacksquare = 0$

We will use this and factoring techniques to solve equations.

2nd degree ⇒ exponent 2

Example: Solve $x^2 - 10x = 0$

*factor the LHS* $x(x-10) = 0$

$F_1$ $F_2$

$\therefore$ *either* $x$ *or* $x-10$ *must be 0.*

*So* $x = 0$ *or* $x - 10 = 0$

$x = 10$

$$x = \{0, 10\}$$

Example: Solve $x^2 - 2x - 15 = 0$

*factor the LHS*

$(x-5)(x+3) = 0$

$F_1 \cdot F_2 = 0$

$x-5=0$ or $x+3=0$

$x = 5$ $\quad x = -3$

$x = \{-3, 5\}$

Example: Solve $-2x^2 - 5x + 3 = 0$

$-6 = m \times n$, $m + n = -5$, $m = -6$, $n = 1$

Factor the *LHS*

$-2x^2 - 6x + 1x + 3 = 0$

$-2x(x+3) + 1(x+3) = 0$

$(x+3)(-2x+1) = 0$

$x + 3 = 0$ or $-2x + 1 = 0$

$x = -3$

$-2x = -1$

$\frac{-2x}{-2} = \frac{-1}{-2} \Rightarrow x = \frac{1}{2}$

$x = \left\{\frac{1}{2}, -3\right\}$

Solve: $2x^2 - x = 6$ $-6$ $-6$

*Make the equation equal to 0, then factor the LHS.*

$-12 = 2x - 6 = m \times n$
$-1 = m + n$
$-4, 3$

$2x^2 - x - 6 = 0$

$2x^2 - 4x + 3x - 6 = 0$

$2x(x-2) + 3(x-2) = 0$

$(x-2)(2x+3) = 0$

$x - 2 = 0$ or $2x + 3 = 0$

$x = 2$ $2x = -3$

$x = -\frac{3}{2}$

$x = \{-1.5, 2\}$

Solve $4x^2 - 36 = 0$

D.O.S.

$+36 \quad +36$

$4x^2 = 36$

$\div 4 \quad \div 4$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

2 roots (+), (-)

$(2x+6)(2x-6) = 0$

$2x+6=0$ or $2x-6=0$

$2x=-6 \qquad 2x=6$

$x=-3 \qquad x=3$

Solve $2x^2 - 50 = 0$

$2x^2 - 50 = 0$
$+50 \quad +50$
$2x^2 = 50$
$\div 2 \quad \div 2$
$x^2 = 25$
$\sqrt{x^2} = \sqrt{25}$
$x = \pm 5$

$2(x^2 - 25) = 0$
$2(x-5)(x+5) = 0$
$x-5=0$ or $x+5=0$
$x=5$ or $x=-5$

$x = \{-5, 5\}$

Solve $5x^2 - 35 = 0$

$5x^2 - 35 = 0$

$+35 \quad +35$

$5x^2 = 35$

$\div 5 \quad \div 5$

$x^2 = 7$

$x = \pm\sqrt{7}$

$5(x^2 - 7) = 0$

$x^2 - 7 = 0$

$+7 \quad +7$

$x^2 = 7$

$x = \pm\sqrt{7}$

$\therefore x = \{-\sqrt{7}, \sqrt{7}\}$

Example: Solve $14x^2 + 28 = 0$

$14x^2 = -28$

$x^2 = -2$

$x = \pm\sqrt{-2}$

$14(x^2 + 2) = 0$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm\sqrt{-2}$

You can't calculate the square root of a negative number.

No Real solution.

Example:

Solve $10x^2 - 4x - 7 = 4x^2 - 11x + 13$

$-4x^2$ $+11x$ $-13$ $-4x^2$ $+11x$ $-13$

make one side equal to 0

$6x^2 + 7x - 20 = 0$

$6x - 20 = m \times n = -120$

$m + n = 7$

$+15, -8$

Factor

$6x^2 - 8x + 15x - 20 = 0$

$2x(3x - 4) + 5(3x - 4) = 0$

$(3x - 4)(2x + 5) = 0$

$3x - 4 = 0$ or $2x + 5 = 0$

$3x = 4$ $\quad$ $2x = -5$

$x = 4/3$ $\quad$ $x = \frac{-5}{2}$

$x = \left\{-\frac{5}{2}, \frac{4}{3}\right\}$

Example: The length of a rectangle is $5cm$ longer than its width. If the area is equal to $150cm^2$, **what is the numerical value of the perimeter of the rectangle?**

*width*: $x$

*length*: $x+5$

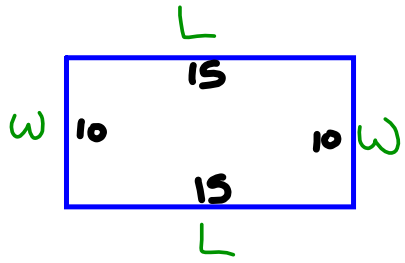
*Area*: $x(x+5)$ or $x^2+5x$

$x^2+5x=150$

$-150 \quad -150$

$x^2+5x-150=0$

$(x+15)(x-10)=0$




$x+15=0$ or $x-10=0$

$x=-15$ or $x=10$

Reject

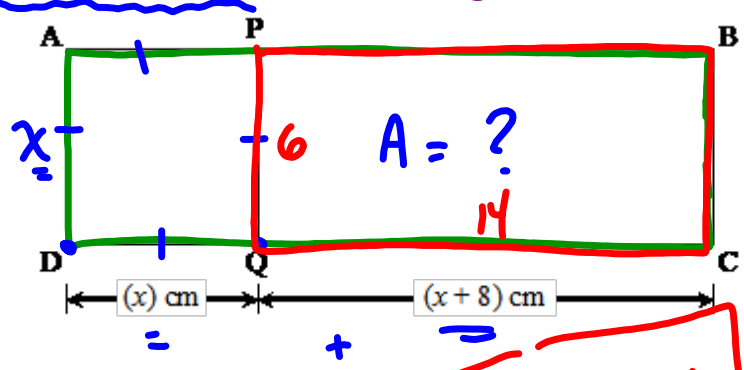
width cannot be (−)

$\therefore x=10$

*Perimeter* = 50 *cm*

Example: In the figure, $\overline{PQ}$ divides rectangle $ABCD$ into two quadrilaterals: square $APQD$ and rectangle $PBCQ$. The area of rectangle $ABCD$ is $120cm^2$. In addition, $mDQ = (x)cm$ and $mQC = (x+8)cm$.

What is the numerical area of rectangle PBCQ?




$L \times W = 120$

$B \cdot H = 120$

$(2x+8)x = 120$

$2x^2 + 8x = 120$

$-120 \quad -120$

$2x^2 + 8x - 120 = 0$

$2(x^2 + 4x - 60) = 0$

$x^2 + 4x - 60 = 0$

$x^2 + 4x - 60 = 0$

$(x+10)(x-6) = 0$

$x + 10 = 0$ or $x - 6 = 0$

$x = -10$ Reject

$x = 6$

Area $= 6 \times 14$

$= 84 cm^2$