Determine the exact value of the y - coordinate of a trigonometric point in quadrant 3, if $P(\theta)=\left(-\frac{15}{17}, y\right)$.

$$\boxed{x^2+y^2=1} \text{ or } \cos^2\theta+\sin^2\theta=1$$

$$\Downarrow$$

$$\left(-\frac{15}{17}\right)^2+y^2=1$$

$$\frac{225}{289}+y^2=1$$

$$y^2=\frac{289}{289}-\frac{225}{289}$$

$$y^2=\frac{64}{289}$$

$$y=\pm\frac{8}{17}$$

$$\therefore y=-\frac{8}{17}$$

Determine the exact value of the x - coordinate of a trigonometric point in quadrant 1, if $P(\theta) = \left(x, \frac{7}{25}\right)$.

$x^2 + y^2 = 1$

$x^2 + \left(\frac{7}{25}\right)^2 = 1$

$x^2 + \frac{49}{625} = \frac{625}{625}$

$x^2 = \frac{625}{625} - \frac{49}{625} = \frac{576}{625}$

$x = \pm \frac{24}{25}$

$\boxed{x = \frac{24}{25}}$?

Q_1 (+,+)

If $\sin\theta = \frac{5}{13}$ where $\frac{\pi}{2} \le \theta \le \pi$, find the exact values of the other 5 ratios.

y

Q₂

(−,+)

$\cos\theta$ (−) $\tan\theta$ (−) $\csc\theta$ $\sec\theta$ (−) $\cot\theta$ (−)

$\frac{\sin\theta}{\cos\theta}$

$x^2 + y^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$x^2 + \left(\frac{5}{13}\right)^2 = 1$

$x^2 + \frac{25}{169} = 1$

$x^2 = \frac{169}{169} - \frac{25}{169} = \frac{144}{169} \Rightarrow x = \pm\frac{12}{13} \Rightarrow$

$\boxed{\cos\theta = -\frac{12}{13}}$

$\boxed{\csc\theta = \frac{13}{5}}$

$\boxed{\sec\theta = -\frac{13}{12}}$

$\tan\theta = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{5}{\cancel{13}} \times -\frac{\cancel{13}}{12} = \frac{-5}{12}$

$\cot\theta = -\frac{12}{5}$

If $\cos\theta = \frac{4}{5}$, where $\underbrace{\frac{3\pi}{2} \le \theta \le 2\pi}_{Q_4}$, find the exact values of the other 5 ratios.

x

$(+,-)$

$\sin\theta = y$

$x^2 + y^2 = 1$

$\frac{16}{25} + y^2 = \frac{25}{25}$

$y^2 = \frac{25-16}{25} = \frac{9}{25}$

$y = \pm\frac{3}{5}$

$\therefore \sin\theta = -\frac{3}{5}$

$\sec\theta = \frac{5}{4}$

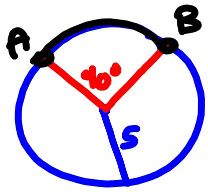
$\csc\theta = -\frac{5}{3}$

$\tan\theta = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4}$

$= -3/4$

$\cot\theta = -4/3$

Arc Length



Recall: $$\frac{\text{central angle}}{360^\circ} = \frac{\text{arc length}}{\text{circumference}}$$

Replacing with radians: $$\frac{\theta}{2\pi} = \frac{arc}{2\pi r}$$

$$\frac{\cancel{2\pi} r\theta}{\cancel{2\pi}} = arc$$

central angle

radius

Arc Length: $L = \theta r$

Note: θ must be in radians.

Examples:

1. Determine the length of the arc, given $r = 12cm$ and $\theta = \frac{2\pi}{3} rad$.

$$L = \theta r$$

$$L = \frac{2\pi}{3} \times 12$$

$$L = \frac{24\pi}{3}$$

$L = 8\pi cm$ or $25.13cm$

2. Determine the diameter of the circle if

$\theta = \frac{5\pi}{6} rad$ and $L = 18cm$. $\Rightarrow$ r

$d = 2 \cdot r$

$$L = \theta r$$

$$18 = \frac{5\pi}{6} r \Rightarrow \frac{18}{\frac{5\pi}{6}} = r$$

$$18 \cdot \frac{6}{5\pi}$$

$$108 = 5\pi r$$

$$\frac{108}{5\pi} cm = r$$

$$\therefore d = \frac{216}{5\pi} cm$$

3. Determine the measure of the central angle, if $L = 36cm$ and $r = 18cm$.

$$L = \theta r$$

$$36 = 18\theta$$

$$2rad = \theta$$

4. Determine the length of the arc (using $L = \theta r$) if $r = 5m$ and $\theta = 400°$.

$$\frac{400}{180} = \frac{\theta}{\pi} \Rightarrow \frac{400\pi}{180} = \theta$$

$$\frac{400\pi}{180} = \theta$$

$$\frac{20\pi}{9} = \theta$$

$$L = \theta r$$

$$L = \frac{20\pi}{9} \times 5$$

$$L = \frac{100\pi}{9} m \quad \text{or} \quad 34.91m$$