Solving Rational Equations

To solve an equation means to determine the possible value(s) of the variables.

To solve rational equations we are going to use all our knowledge about rational expressions and equal fractions (proportions).

Example: Solve for x.

$$\frac{x+5}{x-2}=\frac{3}{8}$$

fraction = fraction
⇒ a proportion

Restriction: $x \neq 2$

$8(x+5) = 3(x-2)$ ⇒ cross-products are equal

$8x + 40 = 3x - 6$
$-3x \qquad -3x$

$5x + 40 = -6$
$-40 \qquad -40$

$5x = -46$

$\frac{5x}{5} = \frac{-46}{5}$

$x = \frac{-46}{5}$

$x = -9\frac{1}{5}$

$x = -9.2$

$-9.2 \neq 2$

$\boxed{x = -9.2}$

Example: Solve for x.

$$\frac{x}{x+2}-\frac{3}{x-2}=1$$

check

$$\frac{-0.4}{1.6}-\frac{3}{-2.4}=1$$

$$-\frac{1}{4}+\frac{5}{4}=1$$

$$-0.25-(-1.25)=1$$

$$\left(\frac{x-2}{x-2}\right)\frac{x}{x+2}-\frac{3}{x-2}\left(\frac{x+2}{x+2}\right)=1 \quad , x\neq\{-2,2\}$$

$$\frac{(x^2-2x)}{(x-2)(x+2)}-\frac{(3x+6)}{(x+2)(x-2)}=1$$

$$\frac{x^2-5x-6}{(x-2)(x+2)}=1$$

$$\frac{x^2 - \overset{m+n}{5x} - \overset{m \times n}{6}}{(x-2)(x+2)} = 1$$

Not a help.

$$\frac{(x+1)(x-6)}{(x-2)(x+2)} = 1$$

$$\frac{x^2 - 5x - 6}{x^2 - 4} = \frac{1}{1}$$

fraction = fraction

$$\cancel{x^2} - 5x - 6 = \cancel{x^2} - 4$$
$$-\cancel{x^2} \qquad\qquad -\cancel{x^2}$$

$$-5x - 6 = -4$$
$$+6 \qquad +6$$
$$-5x = 2$$
$$\frac{-5x}{-5} = \frac{2}{-5}$$

$$x = -\frac{2}{5} \text{ or } -0.4$$

$$-0.4 \neq \{-2, 2\}$$

$$\boxed{x = -0.4}$$

Example: Solve for m.

$$\frac{m-1}{3}-\frac{m+1}{8}=\frac{3m+1}{24}$$

=> fraction = fraction

$$\left(\frac{8}{8}\right)\frac{m-1}{3}-\frac{m+1}{8}\left(\frac{3}{3}\right)=\frac{3m+1}{24}$$

$$\frac{(8m-8)}{24}-\frac{(3m+3)}{24}=\frac{3m+1}{24}$$

$$\frac{5m-11}{24}=\frac{3m+1}{24}$$

Denominators are equal ∴ so are the numerators

$$5m-11=3m+1$$
$$-3m \qquad -3m$$
$$2m-11=1$$

$$2m-11=1$$
$$+11 \quad +11$$
$$2m=12$$
$$\frac{2m}{2}=\frac{12}{2}$$
$$m=6$$

check

$$\frac{5}{3}-\frac{7}{8}=\frac{19}{24}$$

$$\frac{40-21}{24}=\frac{19}{24} \checkmark$$

Example: Solve for a.

$$\frac{1}{a-5}+\frac{1}{a^2-11a+30}=\frac{7}{a-5}$$

m+n m×n

① Factor

$$\frac{1}{a-5}+\frac{1}{(a-5)(a-6)}=\frac{7}{a-5}$$

② Restrictions

$a \neq \{5, 6\}$

③

$$\left(\frac{a-6}{a-6}\right)\left(\frac{1}{a-5}\right)+\frac{1}{(a-5)(a-6)}=\frac{7}{a-5}$$

$$\frac{a-6}{(a-6)(a-5)}+\frac{1}{(a-5)(a-6)}=\frac{7}{a-5}$$

$$\frac{a-5}{(a-6)(a-5)}=\frac{7}{a-5}$$

$$\frac{1}{a-6}=\frac{7}{a-5}$$

$$a-5=7a-42$$

$$-a \qquad -a$$

$$-5=6a-42$$

$$+42 \qquad +42$$

$$\frac{37}{6}=\frac{6a}{6}$$

$$\boxed{a=\frac{37}{6}}$$

Determine the value of x in the diagram below, given that the area of the rectangle is 4 dm^2.

$$\frac{3}{x+1}$$

$$\frac{(2x-5)(x^2+2x+1)}{2x^2-7x+5}$$

$A = 4$

$L \times W = 4$

$2x-5=0$
$2x=5$
$x=5/2$

$2x^2-7x+5$
$m \times n = 10$
$m+n=-7$
$-5, -2$

$2x^2-2x-5x+5$

$2x(x-1)-5(x-1)$

$$\frac{(2x-5)(x^2+2x+1)}{(2x^2-7x+5)} \cdot \frac{3}{x+1} = \frac{4}{1}$$

$$\frac{(2x-5)(x+1)(x+1)}{(x-1)(2x-5)} \cdot \frac{3}{x+1} = 4$$

$x \neq \{-1, 1, \frac{5}{2}\}$

$$\frac{3\cdot\cancel{(2x-5)}\cancel{(x+1)}(x+1)}{(x-1)\cancel{(2x-5)}\cancel{(x+1)}} = 4$$

$$\frac{3(x+1)}{x-1} = \frac{4}{1}$$

$3x+3 = 4x-4$
$-3x \quad -3x$
$3 = x-4$

$\boxed{x=7}$