RATIONAL EXPRESSIONS

A rational expression is a fraction, $\frac{P}{Q}$, where P and Q are polynomials, and $Q \neq 0$.

Examples: $\frac{3x^3+4x-8}{x-6}$, $\frac{5x^2+4}{7}$, $\frac{10}{x}$

A rational expression is undefined for any values of the variables that cause the denominator to be equal to zero.

could be $x-6=0$

Example: $\frac{3x^3+4x-8}{x-6}$

is undefined when x is 6 because $6-6=0$, and we can't divide by 0.

For what values of the variable are the following expressions undefined?

a) $\dfrac{9x^2+4x-7}{x-2}$

$x-2=0$

$\boxed{x=2}$

b) $\dfrac{x-3}{x^2-4}$

$x^2-4=0$

$x^2=4$

$x=\pm 2$

$x=\{-2,2\}$

c) $\dfrac{9}{x^2+4}$

$x^2+4=0$

$x^2=-4$

$x=\pm\sqrt{-4}$

$x=\emptyset$

These values are called restrictions and must be considered when working with rational expressions.

$x \neq 2$ | $x \neq \{-2,2\}$ | No Restriction

Simplifying Rational Expressions [Reducing]

Sometimes a rational expression can be simplified if both the numerator and denominator have a common factor.

Example: $\dfrac{x^2 - 3x}{x^2 - 9}$ - common factor of x

Difference of Squares

Factor the numerator and denominator, if possible.

$$\frac{x(x-3)}{(x+3)(x-3)}$$

$x + 3 = 0$ or $x - 3 = 0$

$x = -3$ $\quad$ $x = 3$

We can cancel out the common factor in the numerator and denominator (the division equals 1), but we must state the restrictions that allow us to do so. $x \neq \{-3, 3\}$

$$\frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \qquad x \neq \{-3, 3\}$$

$$\frac{x}{x+3} \text{ where } x \neq \{-3, 3\}$$

Simplify: factor

a) $\dfrac{a^2-1}{a+1} = \dfrac{(a+1)(a-1)}{(a+1)}$

$a+1=0$
$a=-1$

$a \neq -1$

$\dfrac{\cancel{(a+1)}(a-1)}{\cancel{(a+1)}}$

$a-1, \; a \neq -1$

b) $\dfrac{2x+10}{1x^2+\underbrace{7}_{m+n}x+\underbrace{10}_{m\times n}} = \dfrac{2(x+5)}{(x+5)(x+2)}$

$x+5=0$ or $x+2=0$

$x=-5, -2$

$\therefore x \neq \{-5, -2\}$

$\dfrac{2\cancel{(x+5)}}{\cancel{(x+5)}(x+2)}$

$\dfrac{2}{x+2}, \; x \neq \{-5, -2\}$

c) $\dfrac{v^2 - 7v - 30}{v^2 - 5v - 24}$

$\dfrac{\cancel{(v+3)}(v-10)}{(v-8)\cancel{(v+3)}}$

$v-8=0$ or $v+3=0$

$v \neq \{-3, 8\}$

$\dfrac{v-10}{v-8}$, $v \neq \{-3, 8\}$

factor

d) $\dfrac{6m^3 + 42m^2}{2m^2 + 26m + 84}$

$\dfrac{6m^2(m+7)}{2(m^2 + 13m + 42)}$

$\dfrac{6m^2\cancel{(m+7)}}{2\cancel{(m+7)}(m+6)}$, $m \neq \{-7, -6\}$

$\dfrac{6m^2}{2(m+6)}$

$\dfrac{3m^2}{m+6}$, $m \neq \{-7, -6\}$

Rational Expressions and Arithmetic

To add, subtract, multiply & divide rational expressions, we are going to use the rules of arithmetic for fractions as well as our methods for factoring and simplifying.

1. Multiplication

Example: $\left(\dfrac{3a-3b}{a}\right)\left(\dfrac{a^2}{a-b}\right)$

* factor each polynomial, if possible

$\left(\dfrac{3(a-b)}{a}\right)\left(\dfrac{a^2}{a-b}\right)$

$a \neq 0$

$a-b \neq 0 \Rightarrow a \neq b$

$\therefore a \neq \{0, b\}$

* state the restrictions, then multiply the expressions (canceling any common factors on top and bottom).

$$\frac{3\cancel{(a-b)}a^2}{a^1\cancel{(a-b)}} = a \; , \quad a \neq \{0, b\}$$

$3a$ where $a \neq \{0, b\}$

Example: $\left(\frac{y^2+y}{y-2}\right)\left(\frac{1}{y+1}\right)$

$$\frac{y(y+1)}{(y-2)} \cdot \frac{1}{(y+1)} \quad , \quad y \neq \{-1, 2\}$$

$$\frac{y\cancel{(y+1)}}{(y-2)\cancel{(y+1)}}$$

$$\frac{y}{y-2}, \quad y \neq \{-1, 2\}$$

Example: $\dfrac{x^2-1}{x+3} \times \dfrac{x-3}{1x^2-4x+3}$

$$\frac{(x+1)(x-1)}{(x+3)} \times \frac{(x-3)}{(x-1)(x-3)}, \quad x \neq \{-3, 1, 3\}$$

$$\frac{(x+1)\cancel{(x-1)}\cancel{(x-3)}}{(x+3)\cancel{(x-1)}\cancel{(x-3)}}$$

$$\frac{x+1}{x+3}, \ x \neq \{-3, 1, 3\}$$

2. Division

Example: $$\frac{2x-4}{x^2+6x+9} \div \frac{x^2-4}{x^2-9}$$

* Factor each polynomial, if possible

$$\frac{2(x-2)}{(x+3)(x+3)} \div \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

* State the restrictions. $x \neq \{-3, 3\}$

* Change divide to multiply and flip over the second fraction. State any new restriction(s) this creates.

$$\frac{2(x-2)}{(x+3)(x+3)} \times \frac{(x+3)(x-3)}{(x+2)(x-2)} \quad , \quad x \neq \{-3,-2,2,3\}$$

* multiply, canceling any common factors on top and bottom.

$$\frac{2\cancel{(x-2)}\cancel{(x+3)}(x-3)}{(x+3)\cancel{(x+3)}(x+2)\cancel{(x-2)}}$$

$$\frac{2(x-3)}{(x+3)(x+2)}\text{, where } x \neq \{-3,-2,2,3\}$$