Example: $\dfrac{2x+3}{x+1} - \dfrac{x-4}{x+6}$

$\left(\dfrac{x+6}{x+6}\right)\dfrac{(2x+3)}{x+1} - \dfrac{(x-4)}{x+6}\left(\dfrac{x+1}{x+1}\right)$, $x \neq \{-6, -1\}$

$\dfrac{(2x^2+15x+18)}{(x+6)(x+1)} - \dfrac{(x^2-3x-4)}{(x+6)(x+1)}$

$\dfrac{x^2+18x+22}{(x+6)(x+1)}$ → $m \times n = 22$, $m+n = 18$ ✗

Example: $\frac{a-b}{a^2-1}+\frac{b-1}{a-1}$

Factor wherever possible.

$$\frac{a-b}{(a+1)(a-1)}+\frac{b-1}{a-1}$$

Since there is a common factor in each denominator, we need to multiply one by the factor that will make both look the same.

$$\frac{a-b}{(a+1)(a-1)}+\frac{b-1}{a-1}\left(\frac{a+1}{a+1}\right) \quad ; a \neq \{-1, 1\}$$

State any restrictions.

Multiply, then add the numerators.

$$\frac{a-b}{(a+1)(a-1)}+\frac{ab+b-a-1}{(a+1)(a-1)}$$

$$\frac{ab-1}{(a+1)(a-1)} \quad \text{or} \quad \frac{ab-1}{a^2-1} \qquad a \neq \{-1,1\}$$

Example: $\frac{x}{x^2-9}-\frac{1}{2x-6}$

Factor wherever possible.

$$\frac{x}{(x+3)(x-3)}-\frac{1}{2(x-3)}$$

Make the denominators the same.

Denominators have a common factor

$$\left(\frac{2}{2}\right)\frac{x}{(x+3)(x-3)}-\frac{1}{2(x-3)}\left(\frac{x+3}{x+3}\right), x\neq\{-3,3\}$$

$$\frac{2x}{2(x+3)(x-3)}-\frac{x+3}{2(x-3)(x+3)}$$

$$= \frac{x-3}{2(x+3)(x-3)}, \quad x \neq \{-3, 3\}$$

$$= \frac{1}{2(x+3)}, \quad x \neq \{-3, 3\} \quad \text{or} \quad \frac{1}{2x+6}$$

Example: $\dfrac{x+1}{x^2-2x+1}-\dfrac{1}{x-1}$

$$\frac{x+1}{(x-1)(x-1)}-\frac{1}{x-1}$$

$$\frac{x+1}{(x-1)(x-1)}-\frac{1}{x-1}\left(\frac{x-1}{x-1}\right), \quad x\neq 1$$

$$\frac{(x+1)}{(x-1)(x-1)}-\frac{(x-1)}{(x-1)(x-1)}, x\neq 1$$

$$= \frac{2}{(x-1)(x-1)} \quad \text{or} \quad \frac{2}{x^2-2x+1}, \quad x \neq 1$$