

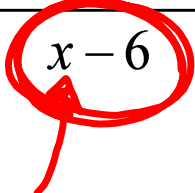
RATIONAL EXPRESSIONS

A rational expression is a fraction, $\frac{P}{Q}$, where P and Q are polynomials, and $Q \neq 0$.

Examples: $\frac{3x^3 + 4x - 8}{x - 6}$, $\frac{5x^2 + 4}{7}$, $\frac{10}{x}$

A rational expression is undefined for any values of the variables that cause the denominator to be equal to zero.

Example: $\frac{3x^3 + 4x - 8}{x - 6}$



is undefined when x is 6 because $6 - 6 = 0$, and we can't divide by 0.

For what values of the variable are the following expressions undefined?

a) $\frac{9x^2 + 4x - 7}{x - 2}$

$$x - 2 = 0$$

$$x = 2$$

b) $\frac{x - 3}{x^2 - 4}$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

c) $\frac{9}{x^2 + 4}$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \sqrt{-4}$$

No Real Answer

These values are called **restrictions** and must be considered when working with rational expressions.

Simplifying Rational Expressions (Reduce)

Sometimes a rational expression can be simplified if both the numerator and denominator have a **common factor**.

Example: $\frac{x^2 - 3x}{x^2 - 9}$

(Handwritten notes: $x^2 - 3x \rightarrow x(x-3)$ and $x^2 - 9 \leftarrow (x+3)(x-3)$)

(Handwritten notes: $(x+3)(x-3)$)

$$\frac{x(x-3)}{(x+3)(x-3)}$$

Factor the numerator and denominator, if possible.

$$x+3=0 \Rightarrow$$

$$x-3=0 \Rightarrow$$

$$\begin{cases} x = -3 \\ x = 3 \end{cases}$$

We can cancel out the common factor in the numerator and denominator (the division equals 1), but **we must state the restrictions** that allow us to do so.

$$\frac{x(x-3)}{(x+3)(x-3)} \quad x \neq \{-3, 3\}$$

$$\frac{x}{x+3} \text{ where } x \neq \{-3, 3\}$$

Simplify:

$$a) \frac{a^2 - 1}{a + 1} = \frac{(a-1)(a+1)}{a+1}$$

$$\left. \begin{array}{l} a+1=0 \\ a=-1 \end{array} \right\}$$

$$\frac{a \neq -1}{\cancel{(a-1)(a+1)}}{\cancel{(a+1)}}$$

$$a \neq -1, a \neq -1$$

$$b) \frac{2x+10}{1x^2+7x+10} \cdot \frac{2\cancel{(x+5)}}{\cancel{(x+2)}\cancel{(x+5)}}$$

\downarrow \downarrow
 $m+n$ $m \times n$
 $2, 5$

$$x+2=0$$

$$x=-2$$

$$x+5=0$$

$$x=-5$$

$$x \neq \{-5, -2\}$$

$$= \frac{2}{x+2}, x \neq \{-5, -2\}$$

$$c) \frac{v^2 - 7v - 30}{v^2 - 5v - 24}$$

$(v - 10)(\cancel{v + 3})$
 $(\cancel{v + 3})(v - 8)$
 $v + 3 = 0 \rightarrow v \neq \{-3, 8\}$
 $v - 8 = 0$
 $\frac{v - 10}{v - 8}, v \neq \{-3, 8\}$

$$d) \frac{6m^3 + 42m^2}{2m^2 + 26m + 84}$$

$\frac{6m^2(m + 7)}{2(m^2 + 13m + 42)}$
 $\frac{6m^2(\cancel{m + 7})}{2(m + 6)(\cancel{m + 7})}, m \neq \{-7, -6\}$
 $m + 6 = 0$
 $m + 7 = 0$
 $\frac{6m^2}{2(m + 6)}, m \neq \{-7, -6\}$
 $= \frac{3m^2}{m + 6}, m \neq \{-7, -6\}$