RATIONAL EXPRESSIONS

A rational expression is a fraction, $\frac{P}{Q}$, where P and Q are polynomials, and $Q \neq 0$.

Examples: $\frac{3x^3+4x-8}{x-6}$, $\frac{5x^2+4}{7}$, $\frac{10}{x}$

A rational expression is undefined for any values of the variables that cause the denominator to be equal to zero.

Example: $\frac{3x^3+4x-8}{x-6}$

is undefined when x is 6 because $6-6=0$, and we can't divide by 0.

For what values of the variable are the following expressions undefined?

a) $\dfrac{9x^2+4x-7}{x-2}$

$x-2=0$

$\boxed{x=2}$

b) $\dfrac{x-3}{x^2-4}$

$(x+2)(x-2)=0$

$x+2=0$

$x-2=0$

$x=-2$

$x=2$

$x^2-4=0$

$x^2=4$

$x=\pm 2$

c) $\dfrac{9}{x^2+4}$

$x^2+4=0$

$x^2=-4$

$x=\sqrt{-4}$ No Real Answer

These values are called restrictions and must be considered when working with rational expressions.

Simplifying Rational Expressions (Reduce)

Sometimes a rational expression can be simplified if both the numerator and denominator have a common factor.

Example: $\dfrac{x^2-3x}{x^2-9}$ $x(x-3)$

D.O.S $(x+3)(x-3)$

Factor the numerator and denominator, if possible.

$$\frac{x(x-3)}{(x+3)(x-3)}$$

$x+3=0 \Rightarrow x=-3$

$x-3=0 \Rightarrow x=3$

We can cancel out the common factor in the numerator and denominator (the division equals 1), but we must state the restrictions that allow us to do so.

$$\frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} \qquad x \neq \{-3, 3\}$$

$$\boxed{\frac{x}{x+3} \text{ where } x \neq \{-3,3\}}$$

Simplify:

a) $\dfrac{a^2-1}{a+1} = \dfrac{(a-1)(a+1)}{a+1}$

$a+1=0$
$a=-1$

$a \neq -1$

$\dfrac{(a-1)\cancel{(a+1)}}{\cancel{(a+1)}}$

$a-1, a \neq -1$

b) $\dfrac{2x+10}{1x^2+7x+10} = \dfrac{2\cancel{(x+5)}}{(x+2)\cancel{(x+5)}}$

m+n m×n
2, 5

$x+2=0$
$x=-2$

$x+5=0$
$x=-5$

$x \neq \{-5,-2\}$

$= \dfrac{2}{x+2}, x \neq \{-5,-2\}$

c) $\dfrac{\overset{m+n}{1v^2 - 7v} \overset{m\times n}{- 30}}{1v^2 \underset{m+n}{-5v} \underset{m\times n}{- 24}}$

$\dfrac{(v-10)\cancel{(v+3)}}{\cancel{(v+3)}(v-8)}$

$v+3=0$

$v-8=0$

$v \neq \{-3, 8\}$

$\dfrac{v-10}{v-8}, v \neq \{-3, 8\}$

d) $\dfrac{6m^3 + 42m^2}{2m^2 + 26m + 84}$

$\dfrac{6m^2(m+7)}{2(m^2 + \underset{m+n}{13m} + \underset{m\times n}{42})}$

$\dfrac{6m^2\cancel{(m+7)}}{2(m+6)\cancel{(m+7)}}, m \neq \{-7, -6\}$

$m+6=0$

$m+7=0$

$\dfrac{6m^2}{2(m+6)}, m \neq \{-7, -6\}$

$= \dfrac{3m^2}{m+6}, m \neq \{-7, -6\}$