d) Factoring a Trinomial

Part 1: The trinomial has the form $ax^2 + bx + c$.

Example: $2x^2 - 3x - 9$

"Product and Sum" Method

1. Multiply the coefficient of the first term and the third term $(a \cdot c)$.

$2x^2 - 3x - 9 \longrightarrow 2 \times (-9) = -18$

2. Find two numbers (m & n) whose sum is the coefficient of the second term (b), and whose product is the value found in step 1 (ac).

1, -18
2, -9
3, -6

$2x^2 - 3x - 9$

$m + n = -3$

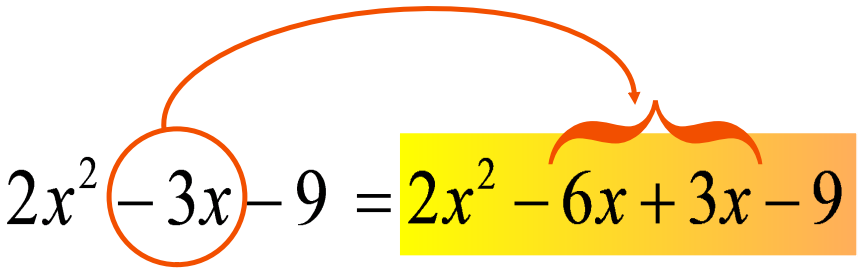
$m \times n = -18$

+, −

$m = -6$

$n = 3$

3. Rewrite the trinomial, replacing the second term with two new terms whose coefficients are the values found in step 2.



4. Factor the new polynomial by grouping.

GCF: $2x$
$F_2(x-3)$

$$2x^2 - 6x + 3x - 9$$

GCF = 3
$F_2 : (x-3)$

$$2x(x-3) + 3(x-3)$$

$$(x-3)(2x+3)$$

$$\therefore\ 2x^2 - 3x - 9 = (x-3)(2x+3)$$

Example: Factor $3x^2+10x+8$

① $a \cdot c = 3 \cdot 8 = 24$

② $m \times n = 24$, $m+n = 10$ } $+6, +4$

③ $3x^2 + 6x + 4x + 8$

④ $3x(x+2)+4(x+2)$

$(x+2)(3x+4)$

Factor the following polynomials.

1. $2x^2 - x - 6$

2. $4x^2 + 12x + 9$

2x 2x·2x·3 3

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① $4 \times 9 = 36$

② $m \times n = 36$, $m + n = 12$ } 6, 6

③ $4x^2 + 6x + 6x + 9$

④ $2x(2x+3) + 3(2x+3)$

$(2x+3)(2x+3)$

$(2x+3)^2$

3. $6x^2 + 11x - 7$

① $6 \cdot -7 = -42$

② $m \times n = -42$, $m + n = 11$ } $+14, -3$

③ $6x^2 - 3x + 14x - 7$

④ $3x(2x-1) + 7(2x-1)$

$(2x-1)(3x+7)$

Factor:

a) $1x^2 - 2x - 48$

① $1 \times -48 = -48$

② $m \times n = -48$, $m + n = -2$ } $+6, -8$

$x^2 + 6x - 8x - 48$

$x(x+6) - 8(x+6)$

$(x+6)(x-8)$

b) $1x^2 - 5x - 14$

① $1 \times -14 = -14$

② $m \times n = -14$, $m + n = -5$ $-7, 2$

$x^2 - 7x + 2x - 14$

$x(x-7) + 2(x-7)$

$(x-7)(x+2)$

Factoring a Trinomial

Part 2: The trinomial has the form $x^2 + bx + c$.

Example: Factor $x^2 + 17x + 60$.

$x^2 + 17x + 60 \Leftrightarrow x^2 + bx + c$ $(a = 1)$

1. Find two numbers (m & n) whose sum is equal to the coefficient of the second term (b), and whose product is equal to the value of the third term (c).

$$\left.\begin{array}{l} m = 5 \\ n = 12 \end{array}\right\} \begin{array}{l} 5 + 12 = 17 \\ 5 \times 12 = 60 \end{array}$$

2. Create the product of two binomials: the first term in each binomial is x (or whatever variable is used); the second term of each binomial are the two values found in step 1.

$$x^2 + 17x + 60 = (x + 5)(x + 12)$$