5. $(5x+4)^2 - 36 = \big((5x+4)+6\big)\big((5x+4)-6\big)$

$\sqrt{(5x+4)^2} = 5x+4$

$\sqrt{36} = 6$

$(5x+10)(5x-2)$

$= 5(x+2)(5x-2)$

6. $(3x+5)^2 - (x-2)^2$

$\big((3x+5)+(x-2)\big)\big((3x+5)-(x-2)\big)$

$(4x+3)(2x+7)$

a) $(3a+7)^2 = (3a)^2 + 2\cdot(3a)(7) + (7)^2 = 9a^2 + 42a + 49$

b) $(2n-3)^2 = 4n^2 - 12n + 9$

factor

$$\underline{(a+b)}^2 = a^2 + 2ab + b^2$$

$$\underline{(a-b)}^2 = a^2 - 2ab + b^2$$

factor

Perfect square trinomials

When given a perfect square trinomial to factor, we can use its characteristics.

Example: $4x^2 + 20x + 25$

$4x^2 + 20x + 25$

this is the square of $2x$

this is the square of 5

this is twice $(2x)(5)$ $= 10x$

$10x \cdot 2 = 20x$

$\therefore$ Factored: $(2x+5)(2x+5)$ or $(2x+5)^2$

The two factors are the same. The terms are the square roots of the first and third terms of the trinomial.

Example: Factor $x^2 + 12x + 36$

$\sqrt{x^2} = x$ $\sqrt{36} = 6$

$2 \cdot x \cdot 6$
$= 12x$ ✓

$(x + 6)^2$

Example: Factor $9x^2 - 12x + 4$

$3x$ $2 \cdot 3x \cdot 2$ 2

$(3x - 2)^2$

Completely Factor $x^4 - 18x^2 + 81$

$\sqrt{x^4} = x^2$ $2 \cdot 9 \cdot x^2$ $18x^2$ $\sqrt{81} = 9$

Difference of squares

$$= \left(x^2 - 9\right)^2$$

$$= \left[(x+3)(x-3)\right]^2$$

or

$$(x+3)^2 (x-3)^2$$

$(a \cdot b)^m$ $a^m \cdot b^m$

d) Factoring a Trinomial

Part 1: The trinomial has the form $ax^2 + bx + c$.

Example: $2x^2 - 3x - 9$

<u>"Product and Sum" Method</u>

1. Multiply the coefficient of the first term and the third term $(a \cdot c)$.

$$2x^2 - 3x - 9 \longrightarrow 2 \times (-9) = -18$$

2. Find two numbers $(m \ \& \ n)$ whose sum is the coefficient of the second term (b), and whose product is the value found in step 1 (ac).

$$m + n = -3$$

$$m \times n = -18$$

$2x^2 - 3x - 9$

$\cancel{1}, \cancel{2}, 3, 6, \cancel{9}, \cancel{18}$

$$\boxed{\begin{aligned} m &= -6 \\ n &= 3 \end{aligned}}$$

3. Rewrite the trinomial, replacing the second term with two new terms whose coefficients are the values found in step 2.

$$2x^2 - 3x - 9 = 2x^2 \overbrace{-6x + 3x} - 9$$

4. Factor the new polynomial by grouping.

Gr 1
GCF: 2x
F_2: (x - 3)

Gr 2
GCF: 3
F_2: (x - 3)

$$\underbrace{2x^2 - 6x} + \underbrace{3x - 9}$$

GCF: (x-3)

$$2x(x-3) + 3(x-3)$$

$$(x-3)(2x+3)$$

$$\therefore \boxed{2x^2 - 3x - 9 = (x-3)(2x+3)}$$

Example: Factor $3x^2 + 10x + 8$

① $a \cdot c = 3 \cdot 8 = 24$

② $\left.\begin{matrix} m \times n = 24 \\ m + n = 10 \end{matrix}\right\} 6, 4$

③ $\underbrace{3x^2 + 6x} + \underbrace{4x + 8}$

④ $3x(x+2) + 4(x+2)$

$(x+2)(3x+4)$

Factor the following polynomials.

1. $2x^2 - x - 6$

$a \cdot c = 2(-6) = -12$

$\left.\begin{matrix} m \times n = -12 \\ m + n = -1 \end{matrix}\right\} -4, +3$

$\underbrace{2x^2 - 4x}\ + \underbrace{3x - 6}$

$2x(x - 2) + 3(x - 2)$

$(x - 2)(2x + 3)$

2. $\underset{2x}{4x^2} + \overset{2 \cdot 2x \cdot 3}{12x} + \underset{3}{9}$

$(2x + 3)^2$

3. $6x^2 + 11x - 7$

$a \cdot c = 6 \cdot -7 = -42$

$\left.\begin{matrix} m \times n = -42 \\ m + n = 11 \end{matrix}\right\} -3, +14$

$= 6x^2 - 3x + 14x - 7$

$3x(2x - 1) + 7(2x - 1)$

$(2x - 1)(3x + 7)$

4. $5x^2 - 35x + 60$

$a \cdot c = 5 \cdot 60 = 300$

$m \times n = 300$, $m + n = -35$ — $-$, $-$

$5(x^2 - 7x + 12)$

$a \cdot c = 1 \times 12 = 12$

$m \times n = 12$, $m + n = -7$ — $-3, -4$

$5(x^2 - 3x - 4x + 12)$

$5[x(x-3) - 4(x-3)]$

$5(x-3)(x-4)$