

b) Factoring by Grouping

Example: Factor $6ab + 3b - 4a - 2$

There is no common factor among all the terms, but some of the terms do share a common factor.

i) Group the terms that have the same common factor.

$$\begin{array}{l} \text{GCF} = 3b \\ \frac{6ab+3b}{3b} = 2a+1 \end{array} \quad \begin{array}{c} 6ab+3b-4a-2 \\ \text{Group 1} \qquad \qquad \text{Group 2} \end{array} \quad \begin{array}{l} \text{GCF} = -2 \\ \frac{-4a-2}{-2} = 2a+1 \end{array}$$

ii) Remove the common factor from each group.

$$\underline{3b(2a+1)} - \underline{2(2a+1)} \quad \text{GCF} = (2a+1)$$

iii) Remove the common factor from both terms

$$(2a+1)(3b-2)$$

Example: Factor $16y^2z - x^2z - 16y^2 + x^2$

$$\begin{aligned} & \underbrace{16y^2z - 16y^2}_{\text{Gr 1}} \quad \underbrace{-x^2z + x^2}_{\text{Gr 2}} \\ & 16y^2(z - 1) - x^2(z - 1) \\ & \underline{\text{GCF:}(z - 1)} \end{aligned}$$

$$(z - 1)(\underbrace{16y^2 - x^2}_{\text{diff of squares}})$$

*This one can
actually be
factored further.

Factor

a) $\underline{xy - x} + \underline{3y - 3}$

$$\begin{aligned} & x(y-1) + 3(y-1) \\ & (y-1)(x+3) \end{aligned}$$

$$\begin{aligned} & \textcircled{1} \quad 2a^3b + 2b^2 + 3a^3 + 3b \\ & 2b(a^3 + b) + 3(a^3 + b) \\ & (a^3 + b)(2b + 3) \end{aligned}$$

$$\begin{aligned} & \textcircled{2} \quad 2a^3b + 3a^3 + 2b^2 + 3b \\ & a^3(2b + 3) + b(2b + 3) \\ & (2b+3)(a^3+b) \end{aligned}$$

c) $\underline{ax + ay + az} + \underline{bx + by + bz}$

$$\begin{aligned} & a(x+y+z) + b(x+y+z) \\ & (x+y+z)(a+b) \end{aligned}$$

d) $12a^2 - 6ab - 8ab + 4b^2$

$$\begin{aligned} & 2(\underline{6a^2} - \underline{3ab} - \underline{4ab} + \underline{2b^2}) \\ & 2[3a(2a-b) - 2b(2a-b)] \\ & 2(2a-b)(3a-2b) \end{aligned}$$