

Factoring Polynomials

To factor a polynomial is to write it as a product.

Example: $5x^2 + 10x$ can be written as $5x(x + 2)$.

a) Removing the Greatest Common Factor

A common factor divides evenly into each term of the polynomial.

Example: Factor $4ab + 6b^2 \Rightarrow 4ab + 6b^2 = F_1 \times F_2$

i) Determine the GCF for the coefficients and the variables.

$4 \div 6$
 2 is the GCF of 4 and 6.
 $b \div b^2$ b is the GCF of ab and b^2 . (always choose the smallest power)

$\therefore 2b$ is the GCF of the polynomial (and the first factor).

ii) Divide the polynomial by the GCF (to get the second factor).

$$\frac{4ab + 6b^2}{2b} = 2a + 3b$$

second factor

iii) Write the polynomial as the product of the two factors.

$$\therefore 4ab + 6b^2 = 2b(2a + 3b)$$

Example: Factor $12x^2y^3 - 27xy^2z$

GCF = $3xy^2$
F₁

$$\frac{12x^2y^3 - 27xy^2z}{3xy^2}$$

F₂: $4xy - 9z$

$$3xy^2(4xy - 9z)$$

Example:

$$\text{Factor } 24x^3y^2 - 8xy^3 + 32x^2y^2z^2$$
$$\text{GCF} = 8xy^2 \quad \frac{24x^3y^2 - 8xy^3 + 32x^2y^2z^2}{8xy^2}$$

$$3x^2 - y + 4xz^2$$

$$8xy^2(3x^2 - y + 4xz^2)$$

Example:

$$\begin{array}{l} \text{Factor } 18a^2b^3c + 27ab^2c^3 + 45abc - 36a^4b^5c^6 \\ 9abc(2ab^2 + 3bc^2 + 5 - 4a^3b^4c^5) \\ \begin{array}{ccc} \text{gcf} & & F_2 \end{array} \end{array}$$

Example: Factor $3a(a-7) + 5(a-7)$

$$\text{GCF} = (a-7)$$

$$\begin{array}{c} \text{term 1} \quad \text{term 2} \\ \hline 3a(a-7) + 5(a-7) \\ \hline (a-7) \\ \hline (3a + 5) : F_2 \end{array}$$

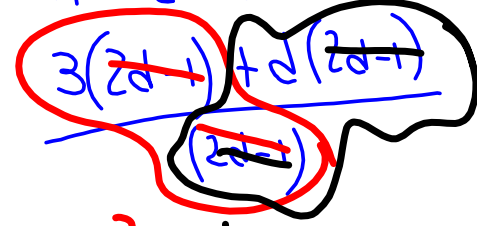
$$(a-7)(3a+5)$$

Factor each of the following:

a) $21xy - 28xyz = 7xy(3 - 4z)$
 GCF = $7xy$

$$\frac{21xy - 28xyz}{7xy} = 3 - 4z$$

b) $18a^2b + 30ab^2c^3$
 $6ab(3a + 5bc^3)$

c) $3(2d-1) + d(2d-1) = (2d-1)(3+d)$
 GCF = $2d-1$

 $3 + d$

$$d) \underline{(b-4)(b+2)} + \underline{(b-5)(b+2)}$$

$$(b+2) = \text{GCF}$$

$$\frac{(b-4)(b+2) + (b-5)(b+2)}{(b+2)}$$

$$(b-4) + (b-5) \text{ or } 2b-9$$

$$(b+2)[(b-4) + (b-5)] = (b+2)(2b-9)$$