Remarkable Identities

Multiply the following binomials.

a) $(x-3)(x+3)$ = $x^2 + 3x - 3x - 9 = x^2 - 9$

b) $(2n+5)(2n-5)$ = $4n^2 - 10n + 10n - 25 = 4n^2 - 25$

c) $(x^2+8)(x^2-8)$ = $x^4 - 8x^2 + 8x^2 - 64 = x^4 - 64$

Did you notice a pattern? Can you formulate a conjecture about the relationship between the terms of the binomials and the product?

$$(a+b)(a-b)=a^2-b^2$$

This is called a difference of squares

these binomials are called conjugates

The product of two binomial conjugates is always a difference of squares.

Use this identity to quickly multiply these conjugates.

a) $(x-13)(x+13)$

$= x^2 - 169$

b) $(5x+12)(5x-12)$

$= 25x^2 - 144$

c) $(100x+15)(100x-15)$

$10000x^2 - 225$

d) $(2x^3+9)(2x^3-9)$

$= 4x^6 - 81$

Square the following binomials.

a) $(x+5)^2 = (x+5)(x+5) = x^2 + 5x + 5x + 25$
$= x^2 + 10x + 25$

b) $(2n-3)^2 = (2n-3)(2n-3) = 4n^2 - 6n - 6n + 9$
$= 4n^2 - 12n + 9$

c) $(p+9)^2 = (p+9)(p+9) = p^2 + 9p + 9p + 81 = p^2 + 18p + 81$

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

This is the square of the second term

This is the square of the first term

This is twice the product of the two terms

Similarly... $(a-b)^2 = a^2 - 2ab + b^2$

The result of squaring a binomial is called a perfect square trinomial.

Use this identity to quickly square these binomials.

a) $(3b-7)^2 = (3b)^2 + 2\cdot(3b)(-7) + (-7)^2$

$= 9b^2 + -42b + 49$

b) $(15x+2)^2$

$225x^2 + 60x + 4$

c) $(17x^4-6y)^2 = 289x^8 - 204x^4y + 36y^2$

$$(101)^2 = (100+1)^2$$
$$= 10000 + 200 + 1$$
$$= 10201$$

$$(99)^2 = (100-1)^2$$
$$= 10000 - 200 + 1 = 9801$$

$$(101)(99) = (100+1)(100-1) = 10000 - 1$$
$$= 9999$$