

Factoring Polynomials

To factor a polynomial is to write it as a product.

$$36 = \underline{6 \times 6} \quad 36 = \underline{9 \times 4} \quad 36 = \underline{12 \times 3}$$

Example: $5x^2 + 10x$ can be written as $5x(x + 2)$.

a) Removing the Greatest Common Factor

A common factor divides evenly into each term of the polynomial.

Example: Factor $4ab + 6b^2$

i) Determine the GCF for the coefficients and the variables.

2 is the GCF of 4 and 6.

b is the GCF of ab and b^2 .

b or b^2
 b ← smallest exponent

$\therefore 2b$ is the GCF of the polynomial (and the first factor).

ii) Divide the polynomial by the GCF (to get the second factor).

$$\frac{4ab + 6b^2}{2b} = \underline{\underline{2a + 3b}} \quad F_2$$

iii) Write the polynomial as the product of the two factors.

$$\therefore 4ab + 6b^2 = 2b(2a + 3b)$$

Example: Factor $12x^2y^3 - 27xy^2z$

$$\text{GCF} = 3xy^2$$

F_1

$$\frac{12x^2y^3 - 27xy^2z}{3xy^2}$$

$(4xy - 9z) F_2$

$$3xy^2(4xy - 9z)$$

Example:

Factor $24x^3y^2 - 8xy^3 + 32x^2y^2z^2$

GCF: $8xy^2$

$$\frac{24x^3y^2 - 8xy^3 + 32x^2y^2z^2}{8xy^2}$$

$$3x^2 - y + 4xz^2$$

$$(8xy^2)(3x^2 - y + 4xz^2)$$

Example:

Factor $18a^2b^3c + 27ab^2c^3 + 45abc - 36a^4b^5c^6$

GCF = $9abc$ $\underbrace{(2ab^2 + 3bc^2 + 5 - 4a^3b^4c^5)}_{F_2}$

F_1 F_2

$$9abc(2ab^2 + 3bc^2 + 5 - 4a^3b^4c^5)$$

Example: Factor $\underbrace{3a(a-7)}_{1 \text{ term}} + \underbrace{5(a-7)}_{1 \text{ term}}$

$$\text{GCF} = (a-7) \quad (F_1)$$

$$\begin{array}{c} \text{3a(a-7) + 5(a-7)} \\ \hline \text{(a-7)} \\ \hline (F_2) \quad 3a + 5 \end{array}$$

$$(a-7)(3a+5)$$

Factor each of the following:

$$\text{a) } 21xy - 28xyz = 7xy(3 - 4z)$$

GCF = $7xy$
 $F_2 = 3 - 4z$

$$\text{b) } 18a^2b + 30ab^2c^3$$

GCF = $6ab$
 $F_2 = 3a + 5bc^3$
 $= 6ab(3a + 5bc^3)$

$$\text{c) } \underline{3(2d-1)} + \underline{d(2d-1)} = (2d-1)(3+d)$$

GCF = $2d-1$
 $F_2 = 3+d$

$$d) \underline{(b-4)(b+2)} + \underline{(b-5)(b+2)}$$

$$\text{GCF: } (b+2)$$

$$(b+2)((b-4) + (b-5))$$

$$\text{Ans: } \underline{(b+2)(2b-9)}$$

$$\frac{(b-4)\cancel{(b+2)} + (b-5)\cancel{(b+2)}}{\cancel{(b+2)}}$$

$$(b-4) + (b-5)$$
$$\underline{\underline{2b-9}}$$