Example: Is $x-\frac{1}{2}$ a factor of $2x^3-9x^2+16x-8$?

Remainder Theorem

$(x-a)$ look like?

$a=\frac{1}{2}$

$2x^3-9x^2+16x-8$

$2\left(\frac{1}{2}\right)^3-9\left(\frac{1}{2}\right)^2+16\left(\frac{1}{2}\right)-8$

$2\left(\frac{1}{8}\right)-9\left(\frac{1}{4}\right)+8-8$

$\frac{1}{4}-\frac{9}{4}+8-8$

$-\frac{8}{4}+8-8$

$-2+8-8=-2$

Remainder = 0? No $r=-2$ ∴ Not a factor

$$\begin{array}{r} 2x^2-8x+12 \\ x-\frac{1}{2}\enclose{longdiv}{2x^3-9x^2+16x-8} \\ -(2x^3-x^2) \\ \hline -8x^2+16x \\ -(-8x^2+4x) \\ \hline 12x-8 \\ -(12x-6) \\ \hline -2 \end{array}$$

Example: Is $(3x-1)$ a factor of $3x^3-x^2-12x+4$?

We have to divide (divisor is not in the form $x-a$).

$$
\begin{array}{r}
x^2 - 4 \\
3x-1\overline{)\,3x^3 - x^2 - 12x + 4} \\
-(3x^3 - x^2) \\
\hline
-12x + 4 \\
-(-12x + 4) \\
\hline
0
\end{array}
$$

Remainder = 0

$\therefore$ $3x-1$ is a factor!

Example: $\left(x^3 - 28x - 41\right) \div (x+4)$

$$
\begin{array}{r}
x^2 - 4x - 12 \\
x+4 \overline{\smash{)}\, x^3 + 0x^2 - 28x - 41} \\
-\left(x^3 + 4x^2\right) \qquad\qquad \\
\hline
-4x^2 - 28x \qquad \\
-\left(-4x^2 - 16x\right) \qquad \\
\hline
-12x - 41 \\
-\left(-12x - 48\right) \\
\hline
7
\end{array}
$$

$x^2 - 4x - 12 + \frac{7}{x+4}$

Work Book, Page 13, Question 3 d, e, f

$$(x^4 - 7x^2 + 8) \div (x + 2)$$

$$x^3 - 2x^2 - 3x + 6$$

$$x+2 \overline{)\, x^4 + 0x^3 - 7x^2 + 0x + 8}$$

$$-(x^4 + 2x^3)$$

$$-2x^3 - 7x^2$$

$$-(-2x^3 - 4x^2)$$

$$-3x^2 + 0x$$

$$-(-3x^2 - 6x)$$

$$6x + 8$$

$$-(6x + 12)$$

$$-4$$

$$x^3 - 2x^2 - 3x + 6 + \frac{-4}{x+2}$$

OR

$$x^3 - 2x^2 - 3x + 6 - \frac{4}{x+2}$$

Example: $\left(x^4+x^3+7x^2-6x+8\right)\div\left(x^2+2x+8\right)$

$$
\begin{array}{r}
x^2 - x + 1 \\
x^2+2x+8 \,\big)\overline{\, x^4 + x^3 + 7x^2 - 6x + 8} \\
-\left(x^4 + 2x^3 + 8x^2\right) \\
\hline
-x^3 - x^2 - 6x \\
-\left(-x^3 - 2x^2 - 8x\right) \\
\hline
x^2 + 2x + 8 \\
-\left(x^2 + 2x + 8\right) \\
\hline
0
\end{array}
$$

Example: $\left(\cancel{-2a^3}\cancel{-10}\cancel{+16a}\cancel{+39a^2}\cancel{-15a^4}\right)\div\left(2-4a-5a^2\right)$

$$3a^2 - 2a - 5$$

$$-5a^2 - 4a + 2 \,\overline{\smash{\big)}\,-15a^4 - 2a^3 + 39a^2 + 16a - 10}$$

$$-\left(-15a^4 - 12a^3 + 6a^2\right)$$

$$10a^3 + 33a^2 + 16a$$

$$-\left(10a^3 + 8a^2 - 4a\right)$$

$$25a^2 + 20a - 10$$

$$-\left(25a^2 + 20a - 10\right)$$

$$0$$