$$\begin{array}{r} 5x^2 - 15x + 41 \\ 2x+5 \overline{\big) 10x^3 - 5x^2 + 7x - 15} \\ -(10x^3 + 25x^2) \\ \hline -30x^2 + 7x \\ -(-30x^2 - 75x) \\ \hline 82x - 15 \\ -(82x + 205) \\ \hline -220 \end{array}$$

$$5x^2 - 15x + 41 + \frac{-220}{2x+5}$$

OR

$$5x^2 - 15x + 41 - \frac{220}{2x+5}$$

$$\begin{array}{r} 3x^2 - 14x + 63 \\ x+4 \overline{\smash{\big)}\, 3x^3 - 2x^2 + 7x + 8} \\ -(3x^3 + 12x^2) \\ \hline -14x^2 + 7x \\ -(-14x^2 - 56x) \\ \hline 63x + 8 \\ -(63x + 252) \\ \hline -244 \end{array}$$

$$\boxed{3x^2 - 14x + 63 - \frac{244}{x+4}}$$

Example: Is $x+2$ a factor of $3x^3+10x^2+x-14$?

To find out, divide... $x+2\overline{\smash{)}3x^3+10x^2+x-14}$

$$\begin{array}{r} 3x^2+4x-7 \\ x+2\overline{\smash{)}3x^3+10x^2+x-14} \\ -(3x^3+6x^2) \\ \hline 4x^2+x \\ -(4x^2+8x) \\ \hline -7x-14 \\ -(-7x-14) \\ \hline 0 \end{array}$$

Remainder is 0!

If the remainder is 0, it is a factor.

$x+2$ is a factor

Remainder Theorem

A polynomial, $P(x)$, is divisible by a binomial $(x-a)$ if and only if $P(a)=0$.

Example: Is $(x-5)$ a factor of $2x^3-4x^2-34x+20$?

$(x-a)=(x-5) \rightarrow \therefore \boxed{a=5} \longrightarrow$ Let $x=5$

$P(5)=2(5^3)-4(5^2)-34(5)+20$

$P(5)=2(125)-4(25)-170+20$

$P(5)=250-100-170+20$

$P(5)=0 \longrightarrow \therefore$ x-5 is a factor.

Example: Is $(x+2)$ a factor of $3x^4 - 6x^2 + 5x - 8$?

$(x-a)$?

$x-a$ $\quad a=-2$

$(x-a) = (x+2)$ or $(x-(-2)) \longrightarrow$ Let $x = -2$

$$P(-2) = 3\left((-2)^4\right) - 6\left((-2)^2\right) + 5(-2) - 8$$

$= 3(16) - 6(4) + 5(-2) - 8$

$= 48 - 24 - 10 - 8$

$= 6$

$6 \neq 0$

No $x+2$ is not a factor.

Example: Is $(3x-1)$ a factor of $3x^3 - x^2 - 12x + 4$?

We have to divide (divisor is not in the form $x-a$).

Remainder = 0?

$$
\begin{array}{r}
x^2 - 4 \\
3x-1 \overline{\smash{)}\, 3x^3 - x^2 - 12x + 4} \\
-(3x^3 - x^2) \\
\hline
-12x + 4 \\
-(-12x + 4) \\
\hline
0
\end{array}
$$

Yes $3x-1$ is a factor.

Example: $\left(x^3 - 28x - 41\right) \div \left(x + 4\right)$

$$
\begin{array}{r}
x^2 - 4x - 12 \\
x+4 \overline{\smash{\big)}\, x^3 + 0x^2 - 28x - 41} \\
-\left(x^3 + 4x^2\right) \qquad\qquad \\
\hline
-4x^2 - 28x \qquad \\
-\left(-4x^2 - 16x\right) \qquad \\
\hline
-12x - 41 \\
-\left(-12x - 48\right) \\
\hline
7
\end{array}
$$

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$$x-2 \overline{\smash{)}\, x^4 - 2x^2 + 10}$$

$$
\begin{array}{r}
x^3 + 2x^2 + 2x + 4 \\
x-2 \overline{\smash{)}\, x^4 + 0x^3 - 2x^2 + 0x + 10} \\
-(x^4 - 2x^3) \\
\hline
2x^3 - 2x^2 \\
-(2x^3 - 4x^2) \\
\hline
2x^2 + 0x \\
-(2x^2 - 4x) \\
\hline
4x + 10 \\
-(4x - 8) \\
\hline
18
\end{array}
$$

Example: $\left(x^4+x^3+7x^2-6x+8\right)\div\left(x^2+2x+8\right)$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x^2+2x+8 & x^4 + x^3 + 7x^2 - 6x + 8 \\
 & -\left(x^4 + 2x^3 + 8x^2\right) \\
\hline
 & -x^3 - x^2 - 6x \\
 & -\left(-x^3 - 2x^2 - 8x\right) \\
\hline
 & x^2 + 2x + 8 \\
 & -\left(x^2 + 2x + 8\right) \\
\hline
 & 0
\end{array}
$$