Remainder Theorem

A polynomial, $P(x)$, is divisible by a binomial $(x-a)$ if and only if $P(a)=0$.

$P(x)$

Example: Is $(x-5)$ a factor of $2x^3-4x^2-34x+20$?

$(x-a)=(x-5) \rightarrow \therefore \boxed{a=5} \longrightarrow$ Let $x=5$

$$P(5)=2(5^3)-4(5^2)-34(5)+20$$

$$P(5)=2(125)-4(25)-170+20$$

$$P(5)=250-100-170+20$$

$x-5$

$P(5)=0 \longrightarrow \therefore$ is a factor.

Example: Is $(x+2)$ a factor of $3x^4-6x^2+5x-8$?

$(x-a)$? yes

$a=-2$

$(x-a)=(x+2)$ or $(x-(-2)) \longrightarrow$ Let $x=-2$

$$P(-2)=3\left((-2)^4\right)-6\left((-2)^2\right)+5(-2)-8$$

$= 3(16)-6(4)+5(-2)-8$

$= 48-24-10-8$

$= 6$

$6 \neq 0$

$\therefore$ x+2 is not a factor

Example: Is $x-\frac{1}{2}$ a factor of $2x^3-9x^2+16x-8$?

$(x-a)$?

$a = \frac{1}{2}$ or 0.5

let $x = \frac{1}{2}$

$2\left(\frac{1}{2}\right)^3 - 9\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) - 8$

$2\left(\frac{1}{8}\right) - 9\left(\frac{1}{4}\right) + 16\left(\frac{1}{2}\right) - 8$

$\frac{1}{4} - \frac{9}{4} + 8 - 8$

$-\frac{8}{4} + 0$

-2

$\frac{2}{1} \cdot \frac{1}{8}$

$= \frac{2}{8}$

$\therefore x - \frac{1}{2}$ is not a factor

Example: What is the remainder of the following polynomial division?

$$x-3\overline{)x^3+3x^2+6x+8}$$

$x-a \Rightarrow a=3$

x^3+3x^2+6x+8
at $x=3$

$= 3^3 + 3(3)^2 + 6(3) + 8$
$= 27 + 27 + 18 + 8$
$= 54 + 26$
$= \underline{\underline{80}}$

$$\begin{array}{r} x^2+6x+24 \\ x-3\overline{)x^3+3x^2+6x+8} \\ -(x^3-3x^2) \\ \hline 6x^2+6x \\ -(6x^2-18x) \\ \hline 24x+8 \\ -(24x-72) \\ \hline \underline{\underline{80}} \end{array}$$

Example: $(x^3 - 28x - 41) \div (x + 4)$

$$
\begin{array}{r}
x^2 - 4x - 12 \\
x+4 \overline{)\, x^3 + 0x^2 - 28x - 41} \\
-(x^3 + 4x^2) \qquad\qquad\quad \\
\hline
-4x^2 - 28x \qquad \\
-(-4x^2 - 16x) \qquad \\
\hline
-12x - 41 \\
-(-12x - 48) \\
\hline
7
\end{array}
$$

Answer: $x^2 - 4x - 12 + \frac{7}{x+4}$

Work Book, Page 13, Question 3 d, e, f

Example: $\left(x^4+x^3+7x^2-6x+8\right)\div\left(x^2+2x+8\right)$

$$
\begin{array}{r l}
 & x^2 - 1x + 1 \\
x^2+2x+8 \big) & x^4 + x^3 + 7x^2 - 6x + 8 \\
 & -\left(x^4 + 2x^3 + 8x^2\right) \\
\hline
 & -1x^3 - 1x^2 - 6x \\
 & -\left(-x^3 - 2x^2 - 8x\right) \\
\hline
 & x^2 + 2x + 8 \\
 & -\left(x^2 + 2x + 8\right) \\
\hline
 & 0
\end{array}
$$

Example: $\left(-2a^3-10+16a+39a^2-15a^4\right)\div\left(2-4a-5a^2\right)$

$$
\begin{array}{r}
3a^2-2a-5 \\
-5a^2-4a+2\,\overline{\big)\,-15a^4-2a^3+39a^2+16a-10} \\
-\left(-15a^4-12a^3+6a^2\right) \\
\hline
10a^3+33a^2+16a \\
-\left(10a^3+8a^2-4a\right) \\
\hline
25a^2+20a-10 \\
-\left(25a^2+20a-10\right) \\
\hline
0
\end{array}
$$