Exponential Growth & Decay Problems

"Start, Keep, Time"

Use: $f(x) = ac^x$

Initial Value

Time

$(1 \pm \text{rate})^*$

1 + rate = growth

1 - rate = decay

Rate is expressed as a per cent

* except when something "doubles" or "triples" etc. Then the base is 2 or 3 etc.

Examples:

1) Today, a loaf of bread costs $4.99. If inflation averages 1.5% per year, then how much will a loaf of bread cost in 50 years?

2) You buy a car worth $24 500. If the depreciation rate of the car is 17% per annum, how much will the car be worth in 5 years?

3) You invest $8 500 in a plan that is expected to return 7% per annum. How long will you have invested your money, if at that time, you have $15 000?

4) A certain strain of bacteria doubles every 6 hours. If a laboratory has 200 of these bacteria today, how many will there be in four days ?

We can use $y = ac^{bx}$ if b is the number of periods in a certain unit of time.

5) A $750g$ piece of radioactive material has a half-life of eight hours. In how many hours will the mass be $30g$?

$$30 = 750\left(\frac{1}{2}\right)^{\frac{x}{8}}$$

$$0.04 = (0.5)^{\frac{x}{8}}$$

$$\log_{0.5} 0.04 = \frac{x}{8}$$

$$\frac{\log 0.04}{\log 0.5} = \frac{x}{8}$$

$$4.6439 = \frac{x}{8}$$

$$37.155 = x$$

Approximately 37 hours

6) When interest is paid n times a year, the value of a certain amount of capital C_0 invested at an annual interest rate r for t years can be calculated as follows:

$$C(t) = C_0\left(1+\frac{r}{n}\right)^{nt}$$

a) What will a $2 500 investment be worth after 5 years if the annual interest rate is 4%, compounded every 3 months?

b) How long will it take a $5 000 investment to triple, if the interest rate is 2.5%, compounded every 6 months?

$$C(t) = C_0\left(1+\frac{r}{n}\right)^{nt}$$

7) A car loses 15 % of its value per year for the first 4 years and 10 % a year for the years that follow. After how many years will a $19 500 car be worth $4 381.78?

8) At 20 years of age, David bought a car whose value was $28 000. The value of the car depreciated by 15% per year and is now worth $12 424.

At Karen's birth, her father invested $2500 in her name at an annual interest rate of 6% compounded monthly. The investment is now worth $13 558.

Which of the two is older and by how much ?

At 20 years of age, David bought a car whose value was $28 000. The value of the car depreciated by 15% per year and is now worth $12 424.

At Karen's birth, her father invested $2500 in her name at an annual interest rate of 6% compounded monthly. The investment is now worth $13 558.

Which of the two is older and by how much ?

9) In 2019, Graham bought a car for $28 600. He predicts that the value of the car will be $17 564 in 2022 and intends to sell it when the value reaches $8000. If the depreciation of the car follows the model of an exponential function, in what year will he sell his car?

10) Many radioactive isotopes are used in medicine for therapies or diagnoses. These isotopes undergo radioactive decay following an exponential curve. Technetium-99m is an isotope used for medical imaging. Its half-life is only 6 hours.
For a scan, a patient has been injected with 20 millicuries (mCi) of Technetium-99m.
After how long, to the nearest hour, will there be only 1% of the initial radioactivity remaining in the patient's body?

A small city is concerned about the growing number of taxis on its streets and decides to look at limiting the number of taxi licences it issues.
At the beginning of 2003, there were 280 taxis in the city and that number is expected to triple every 10 years.
There were 4000 people in the city who used taxis in 2003.
The number of people in the city who use taxis has been increasing by 1% every year.
Let *R* represent the ratio of the number of taxis to the number of people using taxis.

$$R = \frac{\text{number of taxis}}{\text{number of people}}$$

The city has decided that it will limit the number of taxi licenses it issues when $R \geq 0.35$. **In what year will the city begin to limit the number of taxi licenses it issues**?