# Systems of Linear Equations

- Two lines that don't have the same slope must intersect (cross) at one point.
- We call two lines sharing the same plane a "system".
- Finding the point of intersection is called "solving a system".

There are three (3) ways to solve a system.

1. Graph it

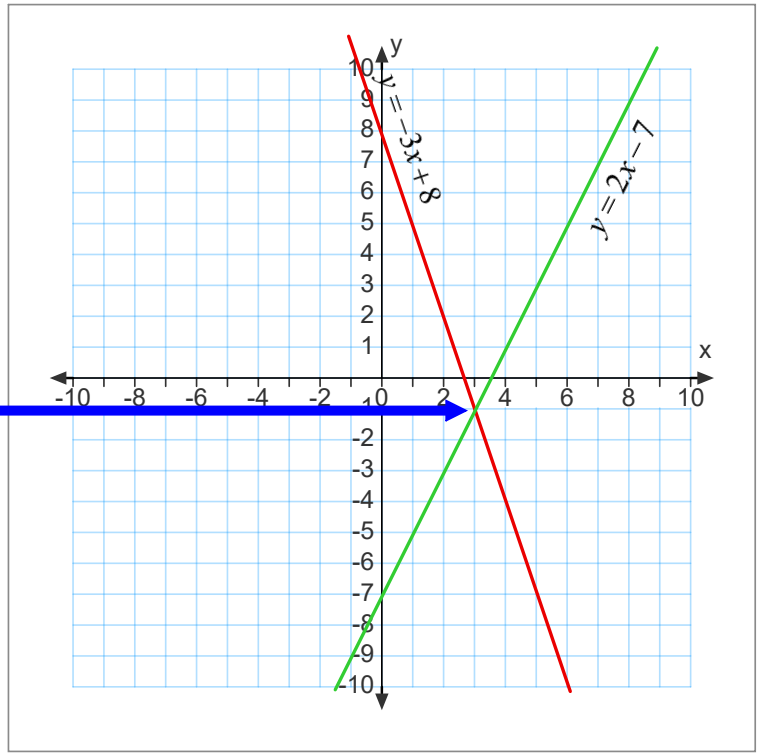
2. Make a table of values

3. Use algebra (3 methods)

Example: Solve the system $y = -3x + 8$

$y = 2x - 7$

1. Graph




Read the point from the graph

Solution: $(3, -1)$

1 $y = -3x + 8$

2 $y = 2x - 7$

## 2. Table of values

For each line, choose several of the same values for $x$. Find the $y$-values for each equation. The solution is found when you get the same $y$-value.

| $x$ | $y_1$ |
|---|---|
| -1 | 11 |
| 0 | 8 |
| 1 | 5 |
| 2 | 2 |
| 3 | -1 |



| $x$ | $y_2$ |
|---|---|
| -1 | -9 |
| 0 | -7 |
| 1 | -5 |
| 2 | -3 |
| 3 | -1 |

Solution: $(3, -1)$

1. $y = -3x + 8$
2. $y = 2x - 7$

## 3. Algebraic Method #1: Comparison

$$-3x + 8 = 2x - 7$$
$$-3x + 3x + 8 = 2x + 3x - 7$$
$$8 = 5x - 7$$
$$8 + 7 = 5x - 7 + 7$$
$$15 = 5x$$
$$\frac{15}{5} = \frac{5x}{5}$$
$$3 = x$$

To find $y$, choose one of the original equations, fill in your $x$ and solve.

$$y = 2(3) - 7$$
$$y = 6 - 7$$
$$y = -1$$

Solution: $(3, -1)$

# Systems of Linear Equations

1) Comparison: best used when both of the equations are in standard form.

Example: Solve the following system using the comparison method.

$$y = -5x + 17$$
$$y = 5x - 9$$

$$-5x + 17 = 5x - 9$$
$$\begin{matrix} +5x & & +5x \end{matrix}$$
$$17 = 10x - 9$$
$$\begin{matrix} +9 & & +9 \end{matrix}$$
$$26 = 10x$$
$$\begin{matrix} \div 10 & & \div 10 \end{matrix}$$
$$2.6 = x$$

$$y = -5x + 17$$
$$y = -5(2.6) + 17$$
$$y = -13 + 17$$
$$y = 4$$

check

$$y = 5x - 9$$
$$y = 5(2.6) - 9$$
$$y = 13 - 9$$
$$y = 4$$

Example: Solve the following system using the comparison method.

$$y = \frac{7}{4}x + 5$$

$$y = \frac{3}{4}x - 3$$

$$\frac{7}{4}x + 5 = \frac{3}{4}x - 3$$

$$\frac{7}{4}x - \frac{3}{4}x + 5 = \frac{3}{4}x - \frac{3}{4}x - 3$$

$$\frac{4}{4}x + 5 = -3 \quad \text{or} \quad x + 5 = -3$$

$$x + 5 - 5 = -3 - 5$$

$$x = -8$$

$$y = \frac{3}{4}(-8) - 3$$

$$y = -6 - 3$$

$$y = -9$$

check: $y = \frac{7}{4}(-8) + 5$

$$y = -14 + 5$$

$$y = -9$$

Solution: $(-8, -9)$

Frank works at a clothing store. His salary is $200 plus 10% of his total sales. Nancy works at another store and makes $250 plus 8% of her total sales.

How much would they have to sell in order to make the same salary?

# Algebraic Method 2: Substitution

Best used when one of the variables is isolated in one equation but not the other. Usually one equation is in standard form and the other is in general form.

Example: $4x-2y+10=0$, $y=4x$

Example: $3x-5y=30$, $x=2y-1$

We substitute what $y$ is equal to in the other equation, then solve the equation.

$$4x-2(4x)+10=0$$
$$4x-8x+10=0$$
$$-4x+10=0$$
$$-4x=-10$$
$$x=2.5$$

Replace the variable in one of the original equations to find the value of the other variable.

$$y=4(2.5)$$
$$y=10$$

Solution: $(2.5,10)$

example: $x = -y - 4$

$3x + 2y = 5$

$3(-y-4)+2y=5$

$-3y-12+2y=5$

$-y-12=5$

$-y=17$

$y=-17$

$x=-(-17)-4$

$x=17-4$

$x=13$

Solution: $(13,-17)$

example: Solve the system $\begin{aligned} 2x + y &= 5 \\ 3x - 2y &= 18 \end{aligned}$

Neither variable is isolated. But in the first equation, we can see that it would be easy to isolate $y$.

So... $\begin{aligned} y &= -2x + 5 \\ 3x - 2y &= 18 \end{aligned}$

Now we can solve using substitution.

$$y = -2x + 5$$
$$3x - 2y = 18$$

$$3x - 2(-2x + 5) = 18$$
$$3x + 4x - 10 = 18$$
$$7x - 10 = 18$$
$$7x - 10 + 10 = 18 + 10$$
$$7x = 28$$
$$x = 4$$

$$y = -2(4) + 5$$
$$y = -8 + 5$$
$$y = -3$$

Solution is $(4, -3)$

Example: A father is 9 years younger than 4 times his daughter's age. The difference of their ages is 27.

How old is each person?

Solve the systems.

1 $y = 5x + 9$

$y = 3x - 12$

2 $y = 4x - 1$

$3x - 2y + 11 = 0$

## Algebraic Method 3: Elimination

Best when both equations are in general form.

example: $x+3y=5$
$7x+6y=20$

1) Choose one of the variables to eliminate ($x$ or $y$).

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$(x+3y=5)\times -7$
$7x+6y=20$

$-7x-21y=-35$
$7x+6y=20$

2) Multiply (if necessary) each equation by the values that will make the coefficients of the chosen variable opposites in each equation.

$$\begin{array}{rl} & -7x-21y=-35 \\ + & 7x+6y=20 \end{array}$$

3) Add the resulting equations together.

$$-15y=-15$$
$$y=1$$

4) Solve the equation for the remaining variable.

$$7x+6(1)=20$$
$$7x+6=20$$
$$7x=14$$
$$x=2$$

5) Take one of the original equations, replace the variable, and solve for the other variable.

Solution: $(2,1)$

example: $-6x-5y+3=0$

$8x+7y=14$

example: $13x + 18y = -273$

$5x + 9y = -186$

Solve each system.

1. $2x+3y=13$
   $x-2y=-4$

2. $3x+4y=29$
   $2x-5y=-19$

Solve:

$$-6x + 2y = -3$$
$$15x + 3y = 51$$

Example: Daisy buys 2 scarves and 3 pairs of jeans for $219.

Mable pays $301 at the same store for 3 scarves and 4 pairs of jeans.

What will Sadie pay for 5 scarves and one pair of jeans?

Solve the following system of equations:

$$2x-3y=-15$$
$$x=1.5y+12$$

by substitution

$$2(1.5y+12)-3y=-15$$
$$3y+24-3y=-15$$
$$24=-15$$

No solution!

When a solution is not possible, this means that the two lines are parallel and distinct.

Solve the following system of equations:

$$20x - 12y = 48$$
$$6y - 10x = -24$$

By elimination

$$20x - 12y = 48$$
$$-10x + 6y = -24 \quad |\times 2$$

$$20x - 12y = 48$$
$$-20x + 12y = -48$$

$0 = 0$ Infinite number of solutions!

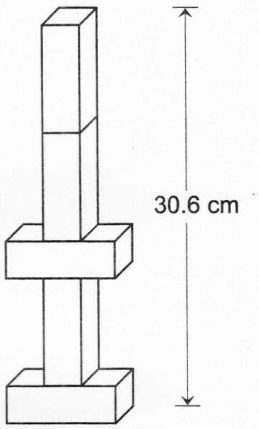
When the system reduces to $0 = 0$, this means the two lines are parallel and coincident (i.e.,they are the same line).

**TOY BLOCKS**

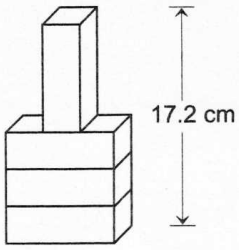
A set of blocks contains several congruent blocks. Each block is a right prism with a square base. These blocks can be stacked horizontally or vertically.

The diagrams below show three stacks of blocks. In addition, the heights of Stack A and Stack B are indicated

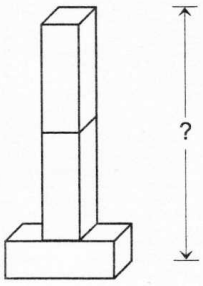
STACK A




STACK B




STACK C




What is the height of Stack C?

**Toy Blocks**

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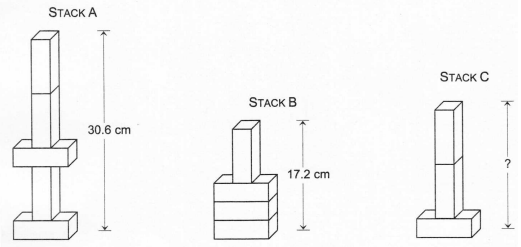


What is the height of Stack C?

Julie, Karina and Veronica made heart-shaped paper clips in two sizes: small and large.
To do so, they cut pieces of wire into two different lengths.

* To make 14 small paper clips and 7 large paper clips, Julie used a total length of 392 cm of wire.
* To make 6 small paper clips and 8 large paper clips, Karina used a total length of 278 cm of wire.
* Veronica made 11 small paper clips and 3 large paper clips.

**What is the total length of wire that Veronica used**?

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