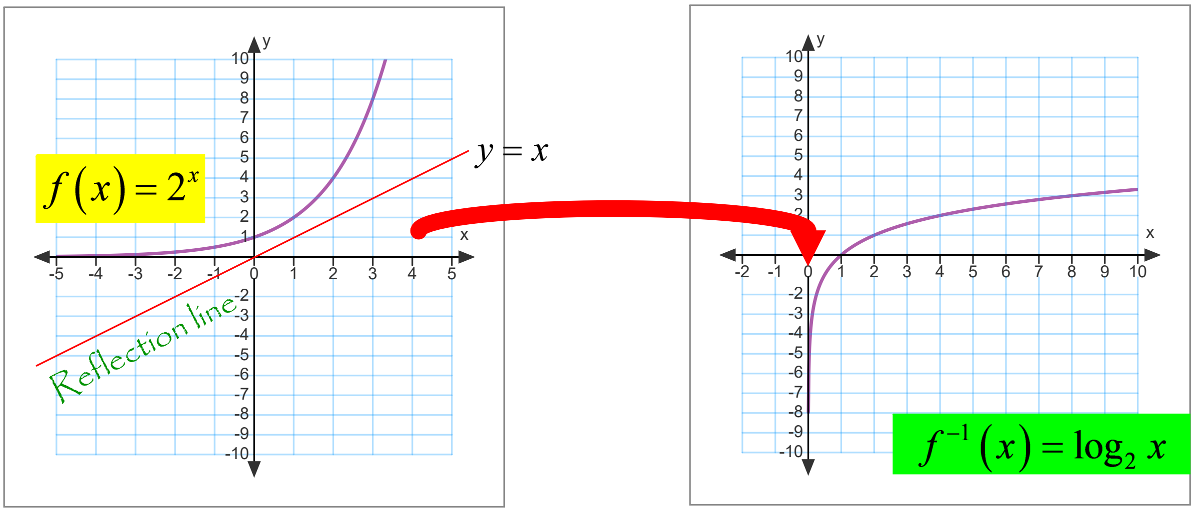
The Logarithmic Function

The inverse of an exponential function is a logarithmic function.



$$y = c^x \longrightarrow x = c^y \longrightarrow y = \log_c x$$

A logarithm is an exponent. It shows the power to which a given base must be raised to produce a given number.

Exponential Form **Logarithmic Form**

$$(base)^{\overset{\text{exponent}}{n}} = \underset{\text{power}}{m} \quad \Longleftrightarrow \quad \log_{(base)} \overset{\text{argument}}{m} = \underset{\text{exponent}}{n}$$

Examples:

1) $2^5 = 32$ ⟷ 1) 🟥

2) 🟩 ⟷ 2) $\log_{16} 4 = \frac{1}{2}$

3) $7^{3x} = 99$ ⟷ 3) 🟦

Convert each of the following to logarithmic form.

1) $5^4 = 625$

[redacted]

2) $12^{-5x} = 506$

[redacted]

3) $81^{\frac{1}{4}} = 3$

[redacted]

Convert each of the following to exponential form.

1) $\log_4 1024 = 5$

[redacted]

2) $\log_{3.6} 12.96 = 2$

[redacted]

3) $\log_9 639 = 2x$

[redacted]

Most calculators use two bases:

1) The common logarithm uses base 10. This base is not usually written.

e.g. $\log_{10}12 = \log\underset{\substack{\uparrow \\ \text{10 understood}}}{} 12$ ⋯⋯→ the log button on the calculator

2) The Napierian or natural logarithm uses base e.

$e \approx 2.71828$. The symbol ln is used.

e.g. $\log_e 20 = \ln 20$

Logarithms using other bases, as in $\log_4 25$, cannot be evaluated directly.

We evaluate using these calculators by performing a conversion called the Change of Base Law.

It says ...

$$\boxed{\log_c m = \frac{\log_n m}{\log_n c}} \Bigg\} \text{ Changing from base } c \text{ to base } n$$

old base

new base

Example: Determine the value of $\log_4 25$.

$$\log_4 25 = \frac{\log 25}{\log 4} \approx \frac{1.39794}{0.60206} \approx 2.322$$

from base 4 to base 10

Determine the values of ...

a) $\log_3 42$

b) $\log_7 135$

c) $\log_4 320$

Basic Logarithmic Function

Rule: $f(x) = \log_c x$

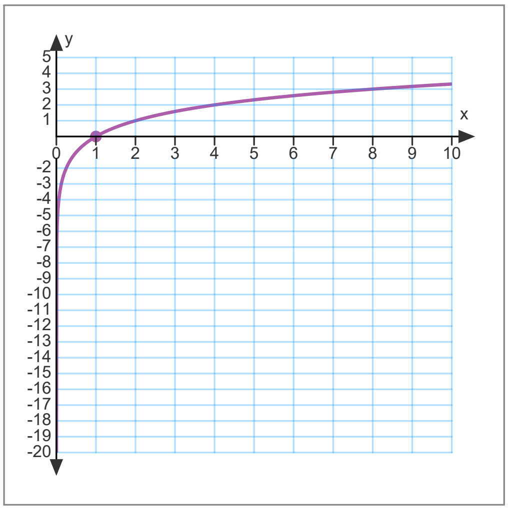
As with the exponential function, the base of the logarithmic function must follow the rules:

$c > 0$ and $c \neq 1$.

Also, the argument, x, must be positive: $x > 0$.

Graphs: Because the basic exponential function has two graphs, so does its inverse, the <u>Basic Logarithmic Function</u> $\left(y = \log_c x\right)$

$c > 1$



y-axis is the asymptote

Properties

Domain: $\left]0, +\infty\right[$

Range: $\mathbb{R}$

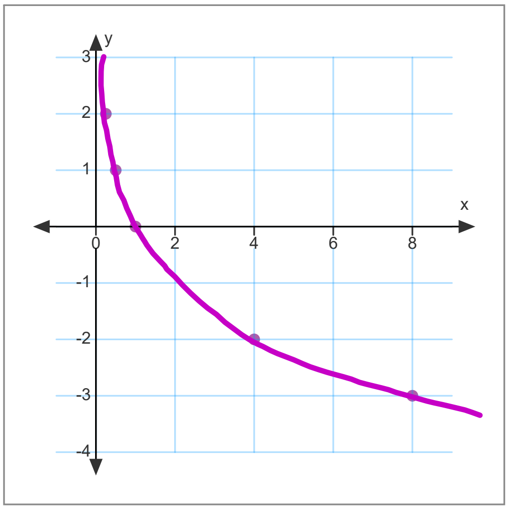
Variation: Increasing over $\left]0, +\infty\right[$

Sign: Neg: $\left]0, 1\right]$ Pos: $\left[1, +\infty\right[$

Extrema: None

Intercepts: Zero = 1

$0 < c < 1$



The y-axis is the asymptote.

Properties

Domain: $]0,+\infty[$

Range: $\mathbb{R}$

Variation: Decreasing over $]0,+\infty[$

Sign: Pos: $]0,1]$ Neg: $[1,+\infty[$

Extrema: None

Intercepts: Zero $= 1$

Transformed Logarithmic Function

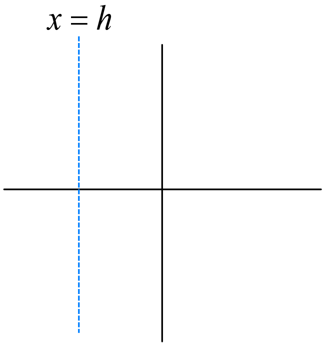
Rule: $f(x) = a\log_c(b(x-h)) + k$

Depending on the sign of a, the sign of b, and the value of c, there are eight (8) different permutations.

$\boxed{c > 1}$

a^+

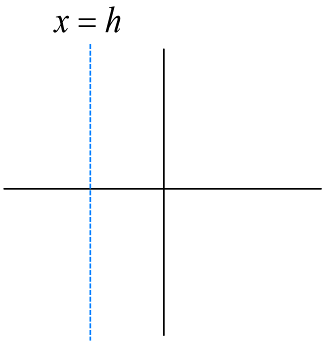
b^+



Increasing, right of asymptote

a^-

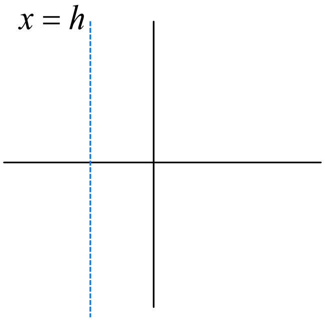
b^+



Decreasing, right of asymptote

a^+

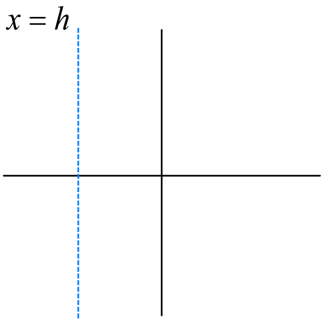
b^-



Decreasing, left of asymptote

a^-

b^-

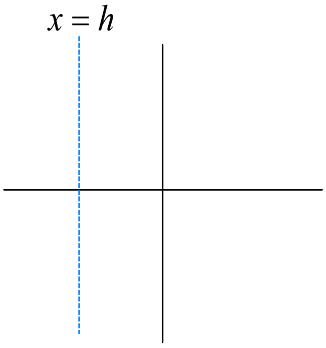


Increasing, left of asymptote

$0 < c < 1$

a^+

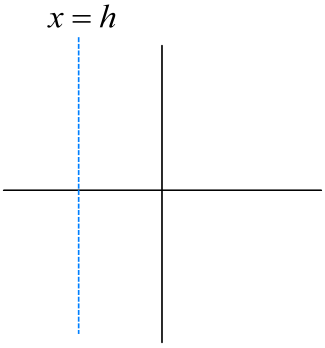
b^+



Decreasing, right of asymptote

a^-

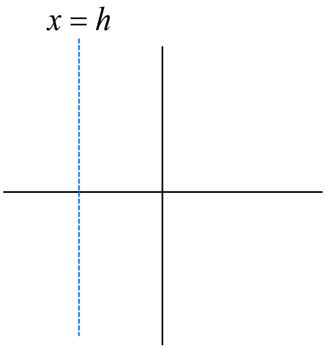
b^+



Increasing, right of asymptote

a^+

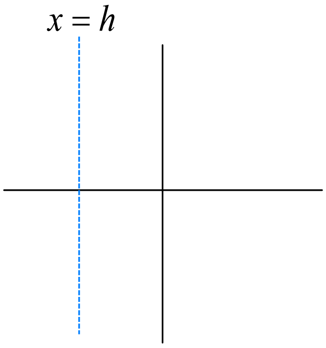
b^-



Increasing, left of asymptote

a^-

b^-



Decreasing, left of asymptote

$x = h$ is the equation of the vertical asymptote.

Finding the Zero

Example: $f(x) = -2\log_{0.5}\left(-3(x-2)\right)+4$

Find the zeros.

a) $f(x)=\frac{2}{5}\log_7\left(0.2(x+7)\right)+1$

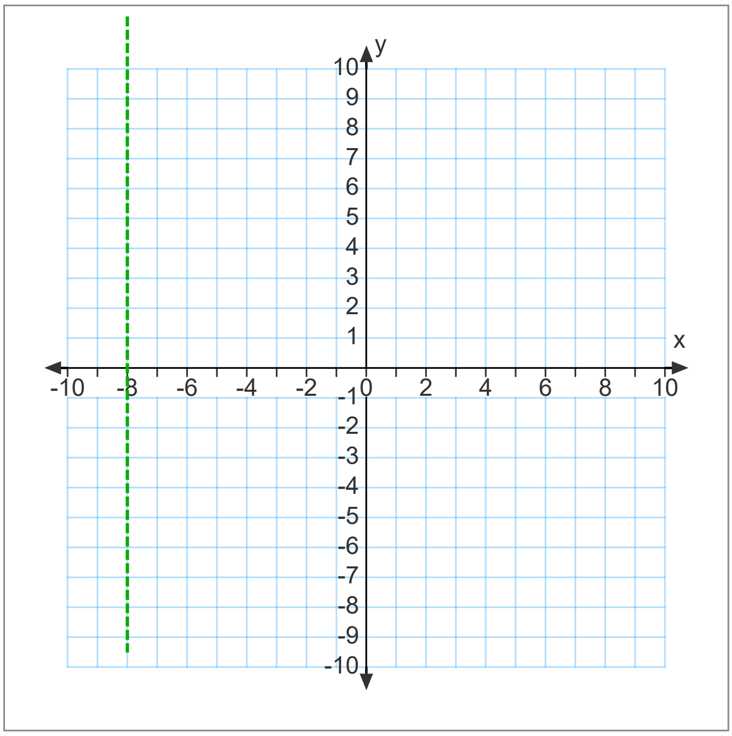
b) $f(x)=9\log_{\frac{3}{4}}\left(\frac{x}{8}-2\right)+3$

Graphing a Logarithmic Function.

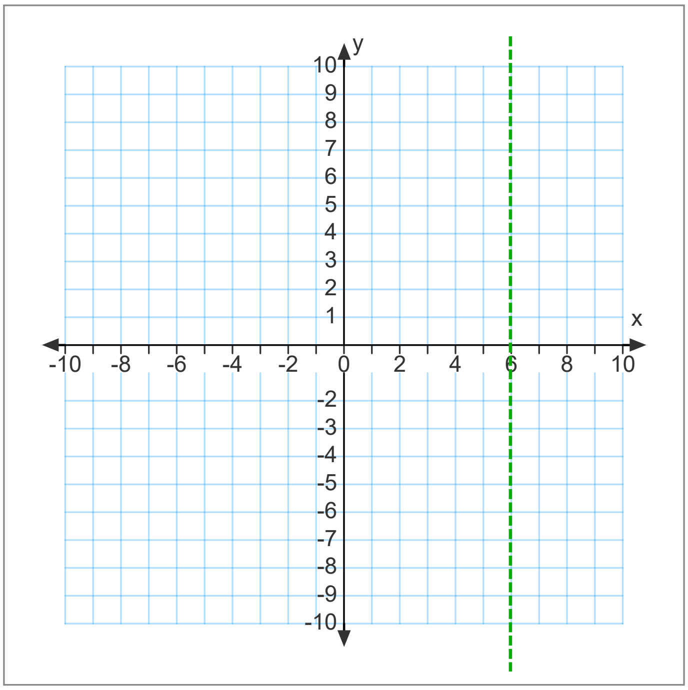
- Locate the vertical asymptote (h).
- Determine the zero (let y =0)
- Draw the curve, based on parameters a, b and c.
- For other points, choose an x value and determine y using the change of base law.

Graph

a) $f(x) = -3\log_2\left(\frac{x}{2}+4\right)+5$



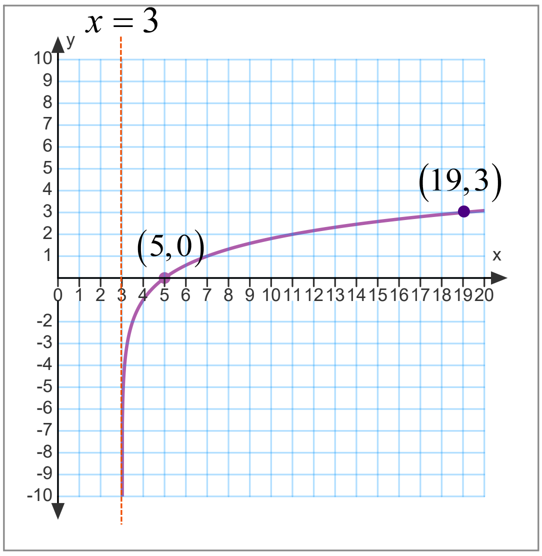
b) $f(x)=5\log_{0.1}\left(-3(x-6)\right)+4$



Finding the Rule of a Logarithmic Function

Use the form: $f(x) = \log_c b(x-h)$

Example:



Zero:

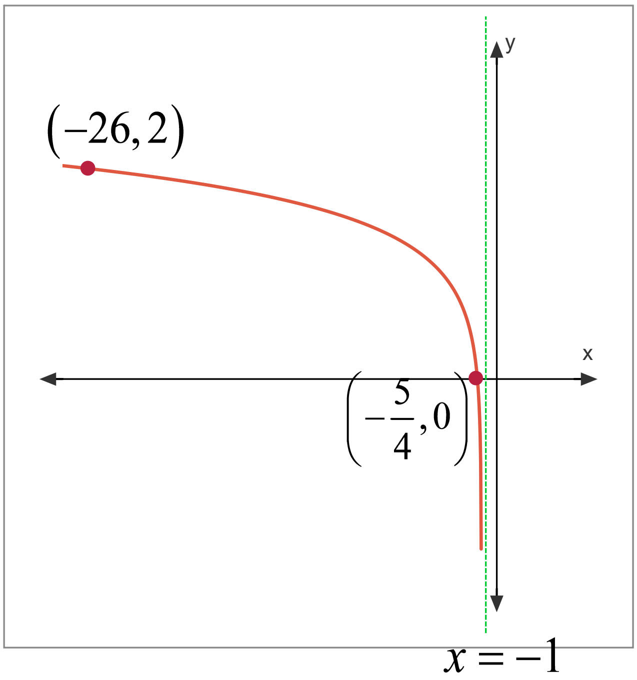
$$0 = \log_c b(x-h)$$

b) Use:

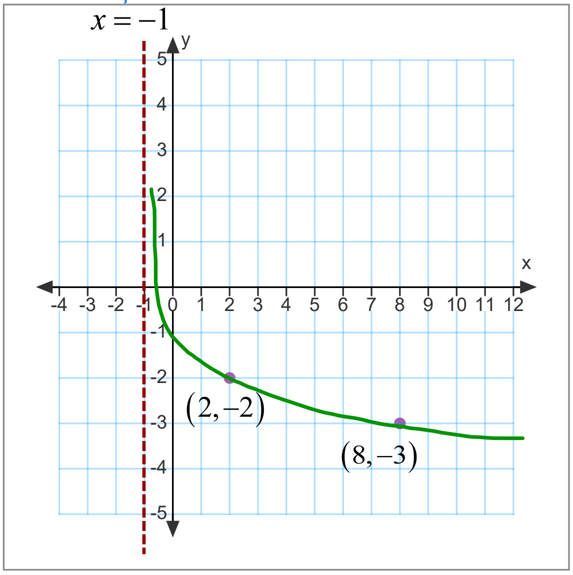
c) Use the other point to find c.

$$f(x)=\log_2\left(\frac{1}{2}(x-3)\right)$$

Example:



Example: Determine the rule.



Solving Equations and Inequalities

Solve for x.

1) $8 = 4(2)^{x-1} + 7$

2) $5 \le -3\left(\frac{1}{5}\right)^{2(x+7)} + 11$

Solve for x.

1) $6\ln 2(x-4)+1=19$

2) $-2\log_5 7(x+5)-8 < -2$

Solve the following inequalities.

a) $4(5)^{0.1x-2}+12\leq 500$

b) $2\log_3(5(x+3))-4\leq 8$

c) $6(4)^{2x} + 9 > 195$

d) $10\log_{16}(-4x+8) - 15 \leq 5$

Determine the equation of the inverse function for...

a) $f\left(x\right)=2(3)^{x-5}+6$ b) $f\left(x\right)=-3\log_8\left(4x+2\right)-7$

Determine the rule of the inverse of the function

$$f(x) = -\frac{1}{2}(3)^{4(x-1)} + 5$$

What is the zero of the inverse of the function

$$f\left(x\right)=-2\log_3\left(-3\left(x-2\right)\right)+4$$