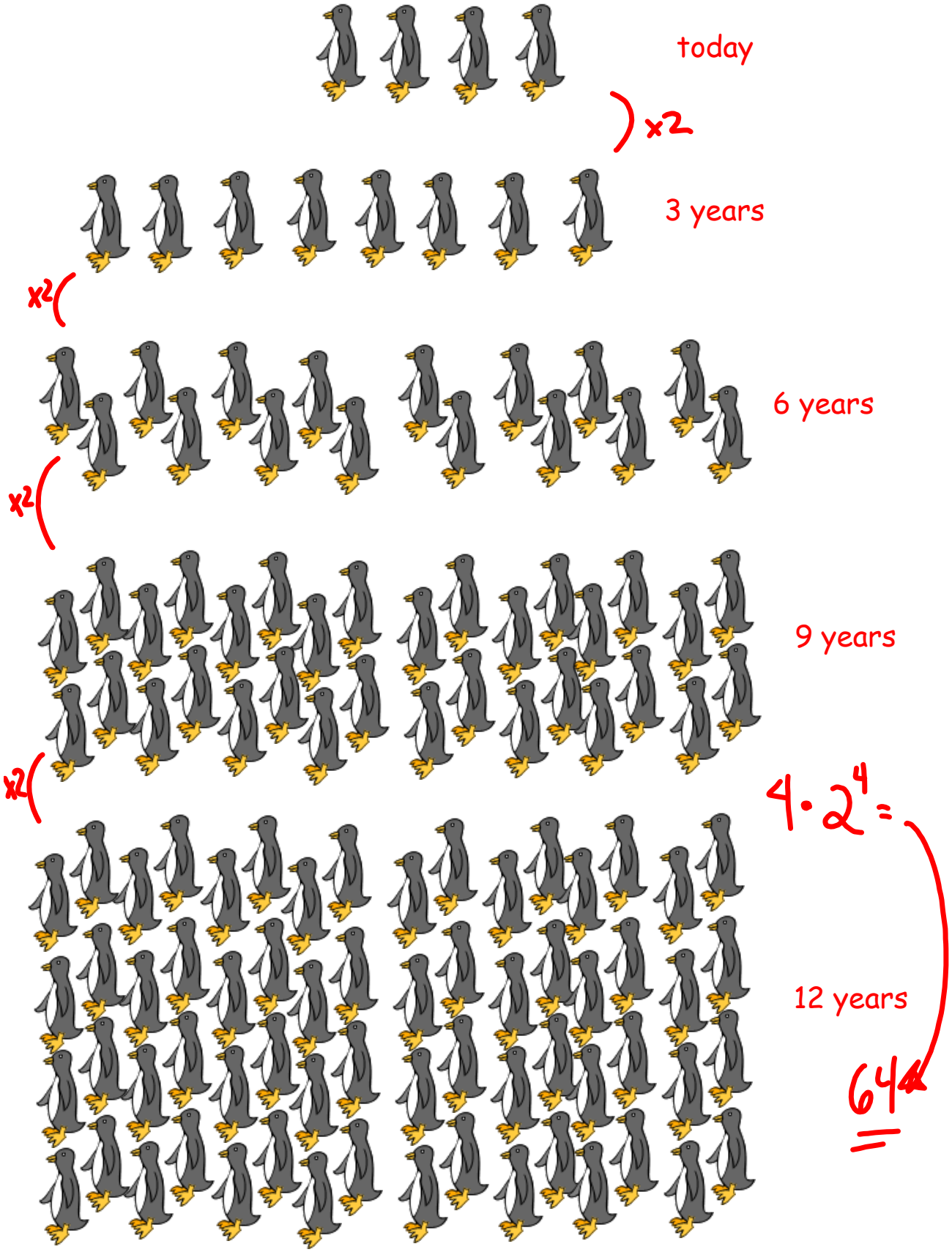
Exponential Function

A population of penguins doubles every 3 years. If a zoo starts with 4 penguins, then how many penguins will they have in 12 years?


today
x2
3 years
x2
6 years
x2
9 years
x2
12 years
$4 \cdot 2^4 =$
64

We started with 4.

That doubled to 8;

then doubled to 16;

then doubled to 32;

then doubled to 64.

This is an example of an exponential function.

$$f(x) = a(base)^x \quad \text{or} \quad f(x) = ac^x$$

There are 3 parts to an exponential function.

We call them (usually)...

1) start
2) keep
3) time

- Start is the value you have at the beginning.
- Keep is how much of the original amount you have (as a percent) after each time period.

  When the amount gets bigger (growth), the keep value is usually (1 + rate of growth).

  When the amount gets smaller (decay), the keep value is usually (1 – rate of decay).

  *** Rate must be converted to a decimal

  *** If the value doubles or triples, then the keep is 2 or 3, etc.
- Time is the number of periods that take place (for example: hours, days, months or years)

Example:

My painting is worth $5200. It appreciates (grows) at an annual rate of ~~4%~~. What will it be worth in 5 years?

use $y = ac^x$

start (a) = 5200 this is the value now

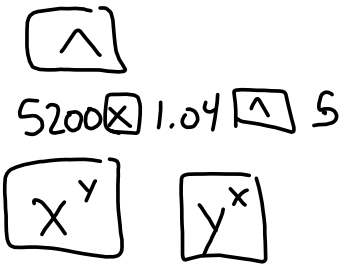
keep (c) = $1 + 0.04 = 1.04$ (base)

time (x) = 5

y = value

$y = 5200(1.04)^5$

$y = \$6326.60$



Example: A couple invests $5 000 for their grand-child's education. It will earn 5% interest each year. What will the investment be worth in 17 years ?

use $y = ac^x$

start = a = 5000

keep = c = 1 + 0.05 = 1.05

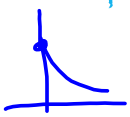
time: x = 17

$y = 5000(1.05)^{17}$

$= \$11\,460.09$

goes down

Example: Shania buys a $24780 car. It depreciates by 12% per year. What will it be worth in 6 years?



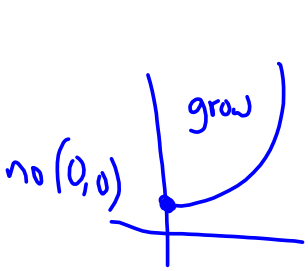
start = a = 24 780

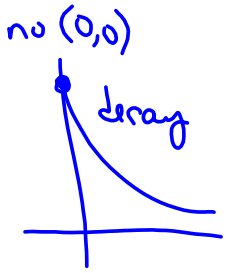
keep = c = 1 − 0.12 = .88

x = time = 6

$y = 24\,780(.88)^6$

$= \$11\,507.93$





Example: Karl bought a car 4 years ago. It has depreciated at a rate of 5% each year since then. Today it is worth $26 064.20. How much did Karl pay for the car 4 years ago ?

$y = 26\,064.20$

$c = 1 - 0.05 = 0.95$

$x = 4$

$a = ?$

$y = ac^x$

$26\,064.20 = a(.95)^4$

$26\,064.20 = 0.8145a$

$\frac{26\,064.20}{0.8145} = a$

$32\,000 = a$

Example: A strain of bacteria triples every hour. If there are 50 bacteria in the lab now, then how long will it take for the culture to grow to 10 000 bacteria ?

use $y = ac^x$

$a = 50$
$c = 3$
$x = ?$
$y = 10\,000$

$10\,000 = 50(3)^x$

guess & check

$200 = 3^x$

x 4.84
y 10.191

x: 4.83
y: 10 060

hour

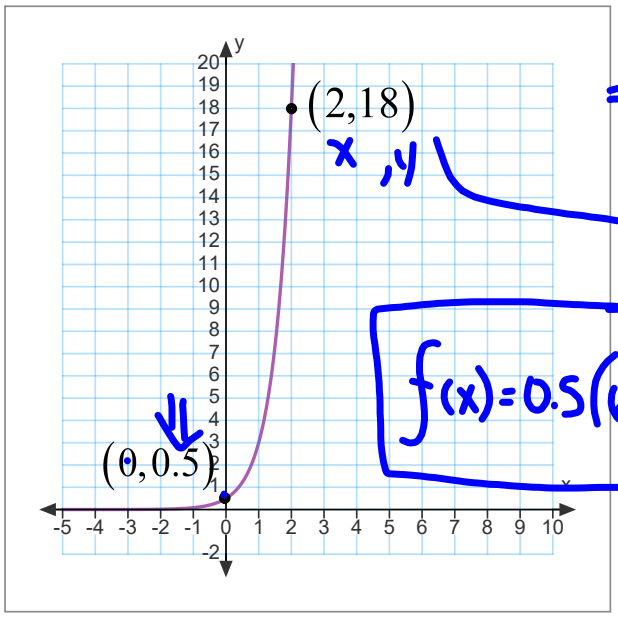
X	Y
10	2 952 450
5	12 150
4	4050
4.5	7014.81
4.8	9753.31
4.9	10 885.9
4.85	10 304.15

Finding the Rule

1) Given the initial value and a point.

Example:

start
y-int



Use the form $f(x) = ac^x$

$\Rightarrow a = 0.5$

c = base = ?

x, y

plug in

$y = 0.5c^x$

$18 = 0.5c^2$

$\div 0.5 \quad \div 0.5$

$36 = c^2$

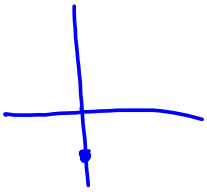
$6 = \sqrt{36} = c$

$f(x) = 0.5(6)^x$

Example: Find the rule

X	Y
-2	-20
-1	-10
0	-5
1	-2.5
2	-1.25

y-int



$y = ac^x$ ← find

① Initial value => y-int

$a = -5$

$(1, -2.5)$

$5^1 = 5$

$1000000^1 = 1000000$

$-2.5 = -5c^1$

$-2.5 \div -5 = -5c \div -5$

$0.5 = c$

$-1.25 = -5c^2$

$0.25 = c^2$

$\sqrt{0.25} = c$

$0.5 = c$

$y = -5(0.5)^x$