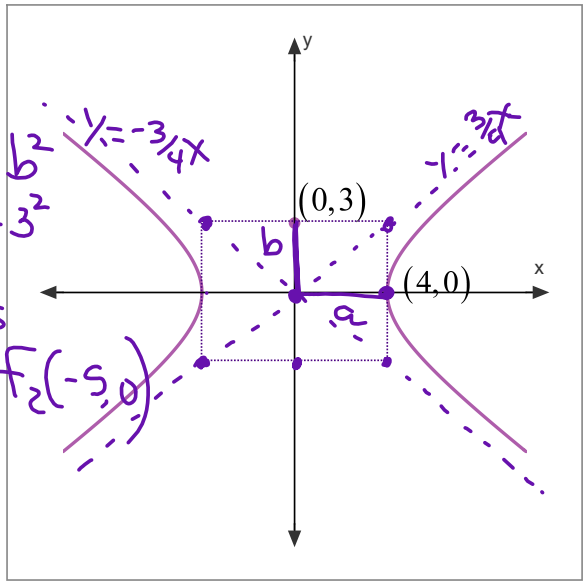
Coordinates of the focus: $c^2 = a^2 + b^2$

(c is the distance from the centre to the focus)

b) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

c) $y = \pm \frac{b}{a}x$ $y = \pm \frac{3}{4}x$

Example:



a) $c^2 = a^2 + b^2$

$c^2 = 4^2 + 3^2$

$c^2 = 25$

$c = \pm 5$

$F_1(5,0)$ $F_2(-5,0)$

a) Find the coordinates of the foci.

b) Find the equation of the hyperbola.

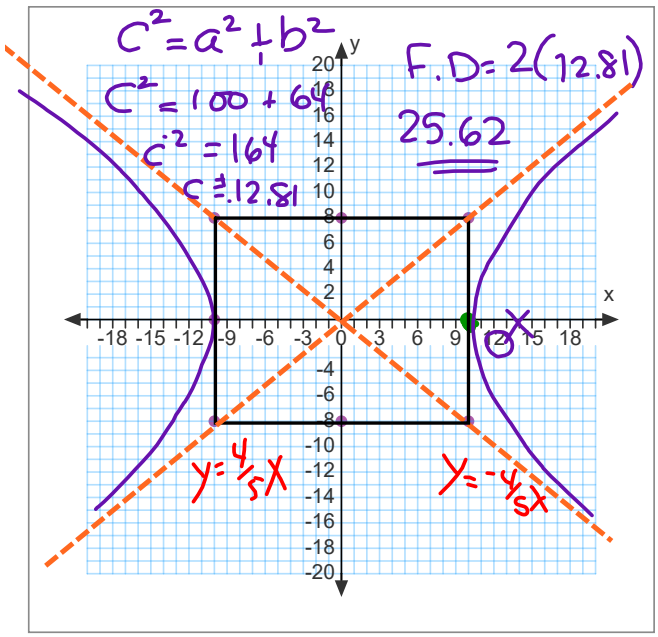
c) Find the equations of the asymptotes.

Example: a) Sketch each hyperbola, including the asymptotes.

b) Determine the focal distance for each hyperbola.

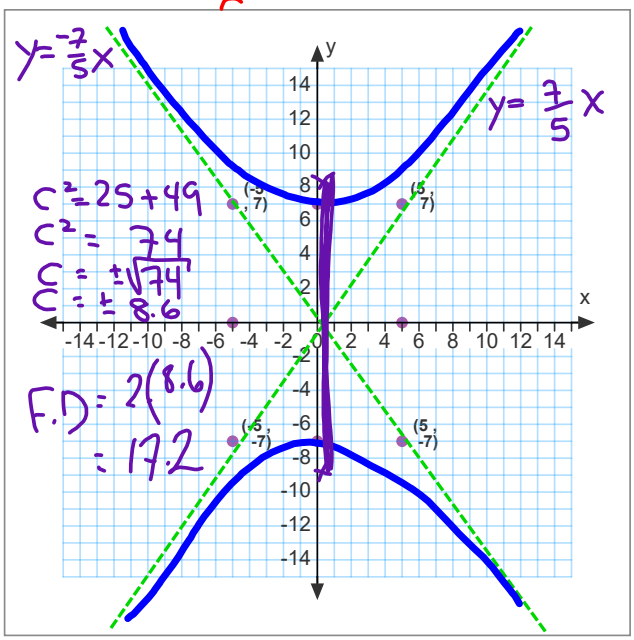
i) $\frac{x^2}{100} - \frac{y^2}{64} = 1$

$a^2 = 100$, $a = \pm 10$; $b^2 = 64$, $b = \pm 8$; Horiz

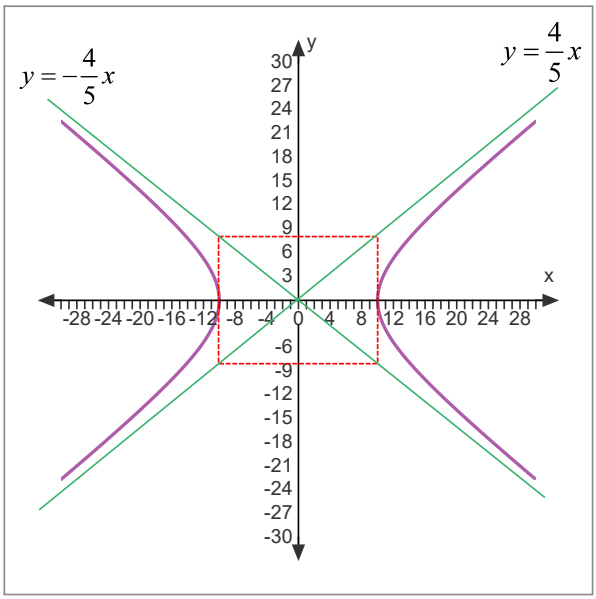


ii) $\frac{x^2}{25} - \frac{y^2}{49} = -1$

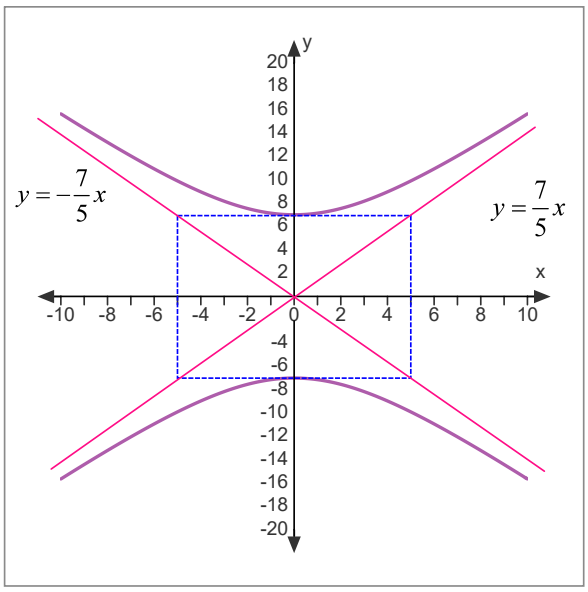
$a = \pm 5$, $b = \pm 7$; vertical; vertices



i)


$y = -\frac{4}{5}x$
$y = \frac{4}{5}x$
y
x
30 27 24 21 18 15 12 9 6 3
-6 -9 -12 -15 -18 -21 -24 -27 -30
-28 -24 -20 -16 -12 -8 -4 0 4 8 12 16 20 24 28

ii)


$y = -\frac{7}{5}x$
$y = \frac{7}{5}x$
y
x
20 18 16 14 12 10 8 6 4 2
-4 -6 -8 -10 -12 -14 -16 -18 -20
-10 -8 -6 -4 -2 0 2 4 6 8 10

Standard to General Form

$\frac{x^2}{49} - \frac{y^2}{9} = 1$ Standard form

$441\left(\frac{x^2}{49} - \frac{y^2}{9} = 1\right)$ Multiply to get rid of the denominators

$\frac{441x^2}{49} - \frac{441y^2}{9} = 441$

$9x^2 - 49y^2 = 441$

$9x^2 - 49y^2 - 441 = 0$ General Form

Convert $\frac{x^2}{4}-\frac{y^2}{25}=-1$ to general form.

$100\left(\frac{x^2}{4}-\frac{y^2}{25}=-1\right)$

$25x^2-4y^2=-100$

$+100 \quad +100$

$25x^2-4y^2+100=0$

General Form: $Ax^2-By^2\pm C=0$

General to Standard Form

$5x^2 - 40y^2 + 160 = 0$ general form

$-160 \quad -160$

$5x^2 - 40y^2 = -160$ Make the RHS equal to ± 1

$160 \quad 160 \quad 160$

$\frac{5x^2}{160} - \frac{40y^2}{160} = -1$ Reduce the fractions

$\frac{x^2}{32} - \frac{y^2}{4} = -1$ Standard Form

Convert $9x^2 - 16y^2 - 144 = 0$ to standard form.

$9x^2 - 16y^2 = 144$

$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$

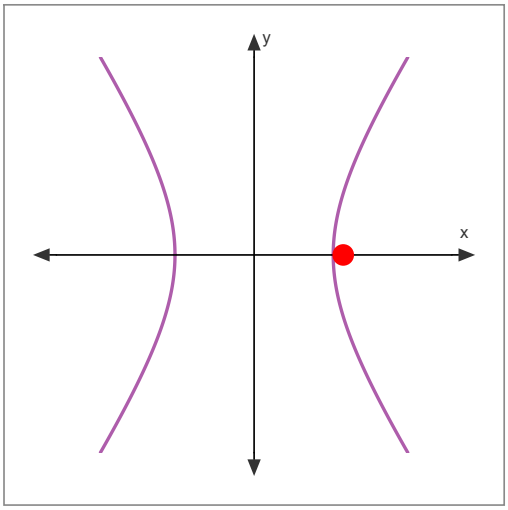
$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Application Problems

1) The lawn of a park covers a region whose edges follow the equation $\frac{x^2}{64} - \frac{y^2}{36} = 1$.

On the right branch, a circular flower bed is to be planted. The endpoints of the diameter of the circle coincide with the focus and vertex of the hyperbola.

What is the equation of this circle?



$\frac{x^2}{64} - \frac{y^2}{36} = 1$

a^2

$a = \pm 8$

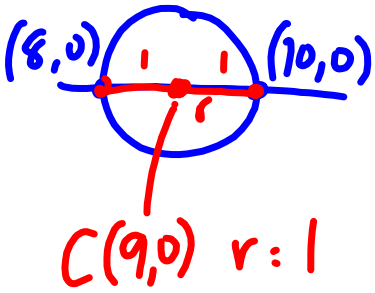
$\therefore V(8,0)$

$c^2 = 64 + 36$

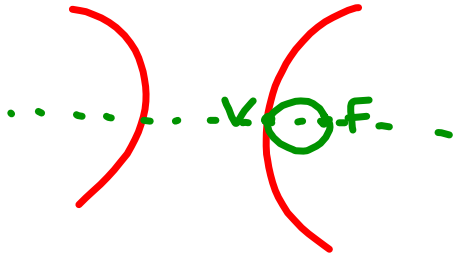
$c^2 = 100$

$c = \pm 10$

$F = (10,0)$



$C(9,0)$ $r: 1$



$(x-9)^2 + y^2 = 1$